Beliefs Heterogeneity and the Equity Term Structure*

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Abstract

What is the role of belief heterogeneity in shaping the equity term structure? We address this question by developing a general equilibrium model featuring habit formation in consumption and heterogeneity in both risk aversion and beliefs about the expected growth rate of the aggregate endowment. We demonstrate that the effects of belief heterogeneity are countercyclical: they increase equity yields during recessions and reduce them during expansions. These effects are more pronounced for short-term assets than for long-term ones. We then examine the role of diagnostic beliefs and show that the overreaction parameter raises equity yields across maturities, with particularly strong effects on short-term maturities during expansions. Overall, our findings highlight the significant influence of belief heterogeneity in shaping the equity term structure.

Keywords: heterogeneous agents; external habit consumption; equity term structure; equity yields
JEL: G11, G12, G51, C63

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1 Introduction

It is well-established in asset pricing literature that beliefs heterogeneity among market participants plays an important role in determining security prices, particularly for long-lived assets (e.g., Anderson et al., 2005; Basak, 2005; Atmaz and Basak, 2018).

However, the impact of belief heterogeneity on assets with varying maturities has received little attention. What role does belief heterogeneity play in pricing assets with different maturities? Do its effects differ between short-term and long-term assets? Are these effects sensitive to changes in the business cycle? This paper addresses these questions by developing a model focusing on the term structure of equity yields.

Our model features an endowment economy with complete markets and "catching up with the Joneses" preferences. The economy is populated by two types of investors who differ in their risk attitudes and beliefs about the expected growth rate of the aggregate endowment. We assume that the conservative investor is also pessimistic, while the bold investor is optimistic.

We begin by demonstrating that modeling habit formation following Chan and Kogan (2002) produces a procyclical slope of equity yields, consistent with empirical evidence (van Binsbergen et al., 2013; Bansal et al., 2021; Giglio et al., 2024). This approach addresses a key shortcoming of the classic habit model proposed by Campbell and Cochrane (1999), which generates a countercyclical slope, shown by Giglio et al. (2024).

Next, we show that belief heterogeneity has significant implications for equity yields. First, the impact of belief heterogeneity depends on the business cycle. In recessions, it raises equity yields across maturities, while in expansions it reduces them. Second, the level effect on equity yields is more pronounced during recessions than expansions. Third, the magnitude of the effect is stronger for short-term assets than for long-term assets.

In recessions, an increase in belief disagreement implies that the bold-optimistic agent assigns a higher probability to states of nature than the conservative-pessimistic agent. Since recessions reflects bad states of nature, the probability of these states are exacerbated by the bold agent when belief disagreement increases. As a result, the risk-sharing rule reduces the consumption share of the bold agent, who is more exposed to risk. Specifically, greater belief disagreement lowers the bold agent's consumption, increasing his marginal utility, which is already high during recessions.

Consequently, the stochastic discount factor decreases because the bold-optimistic agent values current consumption more than future consumption during recessions. This decline in the stochastic discount factor lowers the price of dividend strips, thereby increasing their equity yields. Thus, belief disagreement generates an additional risk premium for assets with different maturities to compensate investors for bearing greater risk during recessions.

In expansions, the opposite occurs. As belief disagreement increases, the bold-optimistic agent assigns a higher probability to states of nature. Since expansions reflects favorable states of the economy, this reinforces his optimistic belief. As a result, the bold agent optimally reallocates resources toward risky assets across maturities, anticipating stronger economic growth than the observed process suggests. Consequently, asset prices across different maturities rise, leading to lower equity yields.

This result contributes to reconciling two strands of literature on the effects of heterogeneous beliefs on a stock's mean return: one suggesting a positive effect (Anderson et al., 2005; David, 2008), and the other indicating a negative effect (Chen et al., 2002; Johnson, 2004). Extending the analysis to assets with different maturities, we find that heterogeneous beliefs positively affect yields during recessions but exert a negative impact during expansions.

Our second result highlights that the effect of belief disagreement on equity yields is more pronounced during recessions than expansions. This outcome is primarily driven by the sensitivity of marginal utility to changes in consumption. Since marginal utility is more responsive at lower consumption levels (recessions) than at higher consumption levels (expansions), an increase in belief disagreement reduces the consumption of the bold-optimistic agent in both economic regimes. However, the reduction in consumption has a stronger impact on marginal utility during recessions, which significantly affects the stochastic discount factor and, consequently, equity yields. Therefore, the effect of belief disagreement on the level of equity yields is stronger during recessions than expansions.

We then incorporate diagnostic beliefs into our framework to examine their impact on equity yields. Diagnostic beliefs capture the tendency of agents to overreact to good or bad news, and we model them as a linear function of the state of the economy, governed by an overreaction parameter. Agents are assumed to be heterogeneous with respect to this parameter. Equipped with this feature, we first compare outcomes under diagnostic beliefs with those under homogeneous beliefs. Our results show that diagnostic beliefs raise equity yields across maturities during expansions and lower them during recessions, with stronger effects on short-term assets in expansions. We then explore the implications of increasing the overreaction parameter for the less risk-averse agent. In this case, equity yields rise across maturities, with particularly strong effects on short-term assets during expansions.

Taken together, our results indicate that heterogeneous beliefs—whether state-independent or of the diagnostic type—have important effects on equity yields. These effects vary across maturities and depend on the state of the business cycle. Thus, differences in belief formation across agents appear to play a critical role in understanding the behavior of equity yields.

This article belongs to the heterogeneous-agent asset pricing literature with focus on heterogeneous beliefs (e.g., Basak, 2005; Bhamra and Uppal, 2014; Atmaz and Basak, 2018). Our model builds on the frameworks of Chan and Kogan (2002) and Bhamra and Uppal (2014), with two key distinctions. First, we use the framework to study the equity term structure, particularly how heterogeneous beliefs influence the procyclicality of its slope. Second, we derive an approximate closed-form solution for equity yields, enabling an analytical exploration of how belief disagreement shapes the slope of the equity term structure. Additionally, this paper relates to the literature on equity term structure (e.g., van Binsbergen et al., 2012, 2013; Callen and Lyle, 2020; Bansal et al., 2021; Schröder, 2024). We contribute to this literature by highlighting the distinct role of belief heterogeneity in shaping the equity term structure across the business cycle—a dimension that has not been explored previously.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 presents the derivation of the equilibrium. Section 4 examines the equity term structure, Section 5 examines diagnostic beliefs, and we conclude in Section 6. Appendix A provides the proofs and derivations, Appendix B details the numerical integration procedure for finding dividend strip prices, and Appendix C describes the step to perform Monte Carlo simulation to derive equity yields under diagnostic beliefs.

2 The Heterogeneous-Agent Economy

In this section, we describe a continuous-time endowment economy populated by two agents who differ in their risk aversion and beliefs. Their preferences feature external habits, following Chan and Kogan (2002).

Uncertainty in the economy is modeled on a filtered probability space $\{\Omega, \mathcal{F}, \mathbf{F}, \mathcal{P}\}$, where a one-dimensional Brownian motion Z is defined. As in Basak (2005), the filtration is augmented to accommodate heterogeneity in agents' priors, thereby introducing belief heterogeneity.

2.1 The Endowment and the Standard of Living Processes

We assume that the exogenous aggregate endowment Y follows a geometric Brownian motion with positive parameters $(Y_0, \mu, \sigma > 0)$, given by:

$$dY_t = \mu Y_t dt + \sigma Y_t dZ_t. \tag{1}$$

This formulation ensures that the endowment remains strictly positive over time. We then define a variable X_t, which represents the standard of living in the economy. Intuitively, X_t captures the average aggregate consumption experienced by the economy in the past. Specifically, it is modeled as the weighted geometric average of past realizations of the endowment Y:

$$X_t = \exp\left(\frac{\int_0^t \chi_s y_s ds}{\int_0^t \chi_s ds}\right), \tag{2}$$

where $y_s = \log Y_s$, and χ_s is a weighting function defined by:

$$\chi_s = \lambda_x e^{-\lambda_x(t-s)}, \quad s \leq t \tag{3}$$

This specification follows Chan and Kogan (2002) and is sufficiently flexible to capture the influence of past endowment realizations on the current standard of living through the decay parameter λ_x. Applying Ito's lemma to Eq. (2), the dynamics of $x_t = \log(X_t)$ are given by:

$$dx_t = \lambda_x(y_t - x_t)dt. \tag{4}$$

Next, we summarize the information contained in Y_t and X_t into a single state variable: the relative (log) consumption,

$$\omega_t = \log(Y_t/X_t), \tag{5}$$

whose dynamics evolve as:

$$d\omega_t = \lambda_x(\bar{\omega} - \omega_t)dt + \sigma dZ_t, \tag{6}$$

where λ_x governs the speed of mean reversion, $\bar{\omega} = (\mu - \sigma^2/2)/\lambda_x$ is the long-run mean, and $\sigma^2/(2\lambda_x)$ is the long-run variance of the process.

Therefore, the relative (log) consumption ω_t serves as the main state variable in our model. One advantage of this formulation is that the dynamics in Eq. (6) enable us to study the economy under recessions ($w_t < \bar{\omega}_t$) and expansions ($w_t < \bar{\omega}_t$) in a unified framework.

2.2 Financial Markets

Financial markets are assumed to be complete, and investment opportunities consist of two long-lived assets: a risky and a riskless asset. We assume that the aggregate dividend D_t is identical to the aggregate endowment, i.e., $D_t = Y_t$. There is only one share of the risky asset, implying that the dividend per share is given by $\delta_t = D_t$.

Let S_t denote the price of the risky asset and B_t the price of the riskless asset. Their dynamics are specified as follows:

$$dS_t = (\beta_t S_t - Y_t)dt + \sigma_t S_t dZ_t, \tag{7}$$

$$dB_t = r_t B_t dt, \tag{8}$$

where β_t denotes the expected rate of return on the risky asset, r_t is the instantaneous risk-free rate, and σ_t is the volatility of the risky return. All of these quantities are endogenously determined in equilibrium as functions of the underlying state variables.

2.3 Agents

Preferences. We assume that the economy is populated by two agents who differ in both their risk aversion and beliefs about the growth rate of the endowment, μ. Their preferences exhibit *external habit formation* as in Abel (1990), Chan and Kogan (2002), and Du (2011). Specifically, each agent $k \in \{1, 2\}$ has a constant relative risk aversion utility function (CRRA) with relative risk aversion coefficient (RRA) γ_k, given by:

$$u(c_{k,t}, X_t) = \frac{(c_{k,t}/X_t)^{1-\gamma_k}}{1-\gamma_k}, \tag{9}$$

where X_t denotes the standard of living in the economy, to which agents seek to adjust. As X_t increases, agents raise their consumption levels accordingly. This complementarity between X_t and current consumption requires the following condition to hold:

$$\frac{\partial^2 u(c_{k,t}, X_t)}{\partial X_t \partial c_{k,t}} = (1-\gamma_k)c_{k,t}^{-\gamma_k} X_t^{\gamma_k - 2} \geq 0, \quad k \in \{1, 2\}.$$

This condition is satisfied when $\gamma_k \geq 1$. Accordingly, we restrict our analysis to values of RRA greater than one. As mentioned in the previous section, X_t is modeled according to Eq. (2).

Beliefs. We also assume that agents have different beliefs about μ, the true expected growth rate of the aggregate endowment. More specifically, agent-k's belief takes the constant value, μ_k. Following the heterogeneous belief literature (Basak, 2005; Bhamra and Uppal, 2014; Atmaz and Basak, 2018), the agent-k's beliefs can be represented by an exponential martingale:

$$\xi_{k,t} = e^{-\frac{1}{2}\sigma_{\xi,k}^2 t + \sigma_{\xi,k} Z_t}, \quad k \in \{1,2\} \tag{10}$$

where the parameter $\sigma_{\xi,k}$ represents the agent-k's beliefs disagreement per unit of risk, expressed as:

$$\sigma_{\xi,k} \equiv \frac{\mu_k - \mu}{\sigma}. \tag{11}$$

The parameter $\sigma_{\xi,k}$ is positive when investor k is more optimistic, and negative when the investor is more pessimistic. Furthermore, $\xi_{k,t}$ denotes the Radon-Nikodym derivative $d\mathcal{P}^k/d\mathcal{P}$, where $\mathcal{P}^k$ represents the subjective probability measure of agent k, and $\mathcal{P}$ is the objective (physical) probability measure. This derivative provides the link between the expectation of a random variable under the agent's subjective measure and its expectation under the physical measure, as follows:

$$\mathbb{E}_0^{\mathcal{P}^k}[F_t] = \frac{\mathbb{E}_0^{\mathcal{P}}[F_t \times \xi_{k,t}]}{\mathbb{E}_0^{\mathcal{P}}[\xi_{k,t}]} = \frac{\mathbb{E}_0^{\mathcal{P}}[F_t \times \xi_{k,t}]}{\xi_{k,0}} = \mathbb{E}_0^{\mathcal{P}}[F_t \times \xi_{k,t}], \quad k = \{1,2\}. \tag{12}$$

The first equality follows from Bayes' theorem, while the second exploits the fact that the Radon-Nikodym derivative $\xi_{k,t}$ is a martingale under the objective probability measure. The third equality assumes $\xi_{k,0} = 1$. When $\xi_{k,t} = 1$, agent-k's beliefs coincide with the objective probability measure, implying $\mu_k = \mu$. Equation (10) can also be expressed as:

$$\frac{d\xi_{k,t}}{\xi_{k,t}} = \sigma_{\xi,k} dZ_t \longrightarrow \xi_{k,t} = \xi_{k,0} \times e^{-\frac{1}{2}\sigma_{\xi,k}^2 t + \sigma_{\xi,k} Z_t}. \tag{13}$$

We then define the *aggregate level of disagreement* between these two investors as[1]

$$\xi_t \equiv \frac{\xi_{2,t}}{\xi_{1,t}} = e^{-\frac{1}{2}(\sigma_{\xi,2}^2 - \sigma_{\xi,1}^2)t + (\sigma_{\xi,2} - \sigma_{\xi,1})Z_t}. \tag{14}$$

When $\xi_t = 1$, both agents have identical subjective probability measures, meaning $\mathcal{P}^1 = \mathcal{P}^2$. We can interpret ξ_t as the ratio of the probability that agent-2 assigns to a particular state relative to the probability assigned by agent-1. The dynamics of ξ_t depends on the difference between the agents' beliefs. Using the baseline calibration (as

[1] Technically, Basak (2005) derives Eq. (14) through the following steps. First, the relationship between an agent's marginal utility and the state price density under the agent's subjective beliefs is established using the first-order conditions of the agent's optimization problem. Second, a link is derived between the agents' weight processes and their marginal utilities by solving the representative agent problem under objective probabilities. Combining these two results yields a relationship between the ratio of agents' weights and the ratio of their state price densities. Finally, applying Itô's lemma to this expression provides the dynamics of the weight ratio, ξ_t.

we show later), agent-1 has optimistic beliefs while agent-2 has pessimistic beliefs, i.e., $\mu_1 > \mu > \mu_2$. If $\xi_t < 1$, this implies $\mathcal{P}^2 < \mathcal{P}^1$. This happens when the economy is hit by positive shocks. Since agent-2 is pessimistic relative to agent-1, she assigns a lower probability to the state than agent-1 does, as a good state of economy is not as likely to happen in her belief. Conversely, if $\xi_t > 1$, pessimistic agent-2 assigns a higher probability to the state of the economy when it is hit by negative shocks.

The dynamics of ξ_t can also be written as

$$\frac{d\xi_t}{\xi_t} = -\sigma_{\xi,1}(\sigma_{\xi,2} - \sigma_{\xi,1})dt + \sigma_\xi dZ_t, \tag{15}$$

where σ_ξ is defined as:

$$\sigma_\xi = \sigma_{\xi,2} - \sigma_{\xi,1} \equiv \frac{\mu_2 - \mu_1}{\sigma}. \tag{16}$$

In this economy, we have two exogenous state variables, ω_t and ξ_t, the relative (log) consumption and aggregate level of disagreement, respectively.

Optimization problem. The portfolio of agent-k is represented by $\left(\omega_{k,t}^{(1)}, \omega_{k,t}^{(2)}\right)$, where $\omega_{kt}^{(1)}$ is the portfolio weight for the risky asset, $\omega_{k,t}^{(2)}$ is the portfolio weight for the riskless asset, and $\omega_{k,t}^{(1)} + \omega_{k,t}^{(2)} = 1$. The agent-$k$'s wealth evolves according to

$$\frac{dW_{k,t}}{W_{k,t}} = \mu_{k,t}dt + \sigma_{k,t}dZ_t, \tag{17}$$

where

$$\mu_{k,t} = \omega_{k,t}^{(1)}(\beta_t - r_t) + r_t - \frac{c_{k,t}}{W_{k,t}} \quad \text{and} \quad \sigma_{k,t} = \omega_{k,t}^{(1)}\sigma_t. \tag{18}$$

Then, we define the stochastic optimal control problem of agent-k for $k \in \{1, 2\}$ as

$$\sup_{\left\{c_{k,t}, \left(\omega_{k,t}^{(1)}, \omega_{k,t}^{(2)}\right)\right\}_{t=0}^{\infty}} E_0^k \left[\int_0^\infty e^{-\rho t} u(c_{kt}, X_t) dt\right], \tag{19}$$

subject to (17) with the given initial value of $W_{k,0}$ and constraints on the control variable $c_{k,t} \geq 0$.

2.4 The Equilibrium

The market equilibrium of the economy is defined by the pair of price processes $\{S_t, r_t\}$ and consumption-trading strategies $\left\{c_{k,t}, \left\{\omega_{k,t}^{(1)}, \omega_{k,t}^{(2)}\right\}; k \in \{1, 2\}\right\}$ such that

1. These strategies solve the stochastic optimal control problem of agent-k for $k \in \{1, 2\}$ described by equations (19), and

2. Markets clear

   - *Goods markets.* The aggregate dividends ($D_t = Y_t$) are used for agents' consumption such that

$$c_{1,t} + c_{2,t} = Y_t \tag{20}$$

- *Financial markets.* There is only one share of the risky asset in the economy (positive net supply), and the riskless asset is in zero net supply.

$$\omega_{1,t}^{(1)} W_{1,t} + \omega_{2,t}^{(1)} W_{2,t} = S_t \quad \text{(risky asset market)} \tag{21}$$

$$\omega_{1,t}^{(2)} W_{1,t} + \omega_{2,t}^{(2)} W_{2,t} = 0 \quad \text{(riskless asset market)} \tag{22}$$

3 Solving for the Equilibrium

In this section, we derive expressions for optimal consumption, the interest rate, the price of risk, and the stochastic discount factor. First, we assign values to the parameters of our model based on previous literature. Second, we use the social planner equilibrium to determine asset prices and equilibrium quantities.

Parameter values. Since our model extends Chan and Kogan (2002) by incorporating heterogeneous beliefs, we adopt their parameter values for the subjective discount rate, ρ, the dynamics of aggregate consumption (μ and σ), and habit persistence, λ_x, as reported in Table 1.

We set the relative risk aversion parameters for the less and more risk-averse agents, γ_1 and γ_2, within the standard range of one to ten commonly used in the macro-finance literature (e.g., Longstaff and Wang, 2012). Specifically, we assume $\gamma_1 = 1.2$ and $\gamma_2 = 2$. In this section, we examine the case in which beliefs are state-independent, meaning that agents remain optimistic or pessimistic regardless of the state of the economy. In this setting, the less risk-averse agent 1 is assumed to be optimistic and holds beliefs about the expected growth rate of the endowment that exceed the true value, i.e., $\mu_1 = \mu + \Delta$. The more risk-averse agent 2, by contrast, holds pessimistic beliefs such that $\mu_2 = \mu - \Delta$. Following Kogan et al. (2006), we set $\Delta = 3\sigma^2 \approx 0.005$. Our baseline calibration aligns with key empirical asset pricing features, including a procyclical price-dividend ratio and countercyclical dynamics in both the price of risk and stock return volatility.

Table 1: Parameter Values

Parameters	Symbol	Values
Subjective discount rate (%)	ρ	5.21
Mean consumption growth (%)	μ	1.8
S.D. consumption growth (%)	σ	4.02
Habit persistence (%)	λ_x	5.87
RRA of less risk-averse agent	γ_1	1.2
RRA of more risk-averse agent	γ_2	2
Weight of agent-1 in social planner	λ	0.66
Belief of less risk-averse agent (optimistic) (%)	μ_1	2.3
Belief of more risk-averse agent (pessimistic) (%)	μ_2	1.3

The Social Planner economy. Given the assumptions of time-separable preferences and complete financial markets, the dynamic consumption-portfolio choice problem simplifies to a sequence of static problems. At each date and state, the planner determines the optimal allocation of consumption between the two investors—that is, the social planner's problem. As shown by Basak (2005), when agents hold heterogeneous beliefs, the weights used to construct the planner's utility function become stochastic processes. We characterize the equilibrium of the social planner's problem to derive each agent's consumption, $c_{k,t} = c_k(\omega_t, \xi_t)$, the interest rate, $r_t = r(\omega_t, \xi_t)$, and the price of risk, $\psi_t = \psi(\omega_t, \xi_t)$, as presented in Lemma 1.

Before presenting the equilibrium results, we define the key objects that summarize heterogeneity in preferences and beliefs: aggregate risk aversion $\mathbf{R}_t$, aggregate prudence $\mathbf{P}_t$, the consumption-share-weighted relative risk tolerance of agent k, $w_{k,t}$, and aggregate belief $\mathbf{u}_t$:

$$\begin{aligned}
\mathbf{R}_t &= \left(\frac{\tilde{c}_{1,t}}{\gamma_1} + \frac{\tilde{c}_{2,t}}{\gamma_2}\right)^{-1}, \\
\mathbf{P}_t &= (1+\gamma_1)\left(\frac{\mathbf{R}_t}{\gamma_1}\right)^2 \tilde{c}_{1,t} + (1+\gamma_2)\left(\frac{\mathbf{R}_t}{\gamma_2}\right)^2 \tilde{c}_{2,t}, \\
w_{k,t} &= \frac{\tilde{c}_{k,t}}{\gamma_k}\mathbf{R}_t, \\
\mathbf{u}_t &= w_{1,t}\mu_1 + w_{2,t}\mu_2.
\end{aligned}$$

Lemma 1. *A Social Planner equilibrium can be constructed under the assumptions of complete financial markets and the absence of arbitrage opportunities. Under these conditions, the following results hold:*

- *The first-order condition (FOC) of the social planner's problem is given by*

$$\lambda_{1,t} u_{c_{1,t}}(c_{1,t}, X_t) = \lambda_{2,t} u_{c_{2,t}}(c_{2,t}, X_t), \tag{23}$$

  where $\lambda_{k,t} \equiv \lambda_{k,0}\xi_{k,t}$ denotes the stochastic weight of agent k's utility in the planner's objective function, $\lambda_{k,0}$ represents agent k's initial endowment, and $u_{c_{k,t}}(c_{k,t}, X_t) = \partial u(c_{k,t}, X_t)/\partial c_{k,t}$ denotes the marginal utility of consumption for agent k, with $k \in \{1, 2\}$. Equation (23) characterizes the optimal risk-sharing condition in the economy, which determines how aggregate consumption is allocated between the two agents in equilibrium. It can be rewritten as:

$$\lambda_{1,0}\xi_{1,t}e^{-\rho t}\left(\frac{1}{X_t}\right)^{1-\gamma_1} c_{1,t}^{-\gamma_1} = \lambda_{2,0}\xi_{2,t}e^{-\rho t}\left(\frac{1}{X_t}\right)^{1-\gamma_2} c_{2,t}^{-\gamma_2}. \tag{24}$$

- *Defining m_t as the equilibrium stochastic discount factor (SDF), and $\tilde{c}_{k,t} \equiv \frac{c_{k,t}}{Y_t}$ as the agent's consumption share, equation (24) can be expressed as:*

$$m_t \equiv \hat{m}_{1,t}\tilde{c}_{1,t}^{-\gamma_1} = \hat{m}_{2,t}\tilde{c}_{2,t}^{-\gamma_2}, \tag{25}$$

  where $\hat{m}_{k,t}$, defined below, represents the SDF under the physical probability measure $\mathcal{P}$ when agent k is the sole representative agent in the economy:

$$\hat{m}_{k,t} = \lambda_{k,0}\xi_{k,t}e^{-\rho t}e^{-\gamma_k\omega_t - x_t}. \tag{26}$$

- *The equilibrium risk-free interest rate is given by:*

$$r_t = \rho + \mathbf{R}_t\mathbf{u}_t - \frac{1}{2}\mathbf{R}_t\mathbf{P}_t\sigma^2 - (\mathbf{R}_t - 1)\lambda_x\omega_t + \frac{1}{2}w_{1,t}w_{2,t}\left(1 - \frac{\mathbf{R}_t}{\gamma_1\gamma_2}\right)\sigma_\xi^2 - w_{1,t}w_{2,t}\mathbf{R}_t\left(\frac{1}{\gamma_1} - \frac{1}{\gamma_2}\right)(\mu_1 - \mu_2), \tag{27}$$

- *The equilibrium price of risk is given by:*

$$\psi_t = \mathbf{R}_t\sigma + \frac{\mu - \mathbf{u}_t}{\sigma}. \tag{28}$$

Proof. See Appendix A.1.

This lemma illustrates the effects of belief heterogeneity on the interest rate and the price of risk. Specifically, belief heterogeneity influences the interest rate through three mechanisms. First, beliefs are aggregated into $\mathbf{u}_t$, increasing or decreasing r_t relative to the homogeneous-belief case, depending on the relative importance of optimistic or pessimistic agents in the economy. Second, differences in beliefs introduce an additional source of uncertainty, which is priced into the interest rate and captured by σ_ξ^2, increasing the interest rate when $\gamma_k > 1$. This effect persists even when agents are homogeneous in their degree of risk aversion. However, when agents also differ in their risk attitudes, belief heterogeneity amplifies this effect, as shown in the last term of Eq. (27). In particular, when the less risk-averse agent ($\gamma_1 < \gamma_2$) is also the more optimistic one ($\mu_1 > \mu_2$), the interest rate decreases.

To gain further insights into the effects of belief heterogeneity on equilibrium quantities, we plot the consumption share of agent 1 as a function of ω_t and ξ_t in Figure 1. It is worth noting that the consumption share of the less risk-averse agent is procyclical; that is, it increases with ω_t. Intuitively, lower risk aversion induces the agent to take on greater exposure to aggregate risk by investing more heavily in risky assets. As a result, her consumption rises in good states of the economy (e.g., when ω_t is high). This mechanism, commonly referred to as risk-sharing in the heterogeneous-agent literature, lies at the core of the determination of equilibrium quantities. Meanwhile, belief disagreement modifies the risk-sharing mechanism: specifically, a lower ξ_t significantly increases the consumption share of agent 1. The intuition behind this result is as follows: when ξ_t is low (e.g., $\xi_t = \frac{\xi_2}{\xi_1} < 1$), the agent 1 assigns more likelihood to state of the world, based on her beliefs. For instance, when the current state is a recession, the agent 1 is surprised since she thought that this state

Next, we plot the equilibrium interest rate and price of risk as functions of ω_t and ξ_t in Figure 2. The risk-free interest rate is negatively related to ω_t because agents exhibit habit preferences, resulting in a countercyclical interest rate. Belief differences affect the sensitivity of the interest rate to the state of the economy by altering its slope. A similar effect appears in the price of risk: a higher ξ_t significantly increases the price of risk, which is also countercyclical, consistent with the existing literature. As the economy becomes more risk-averse in bad states, the price of risk rises, reflecting greater compensation required to bear risk due to heightened aggregate risk aversion.

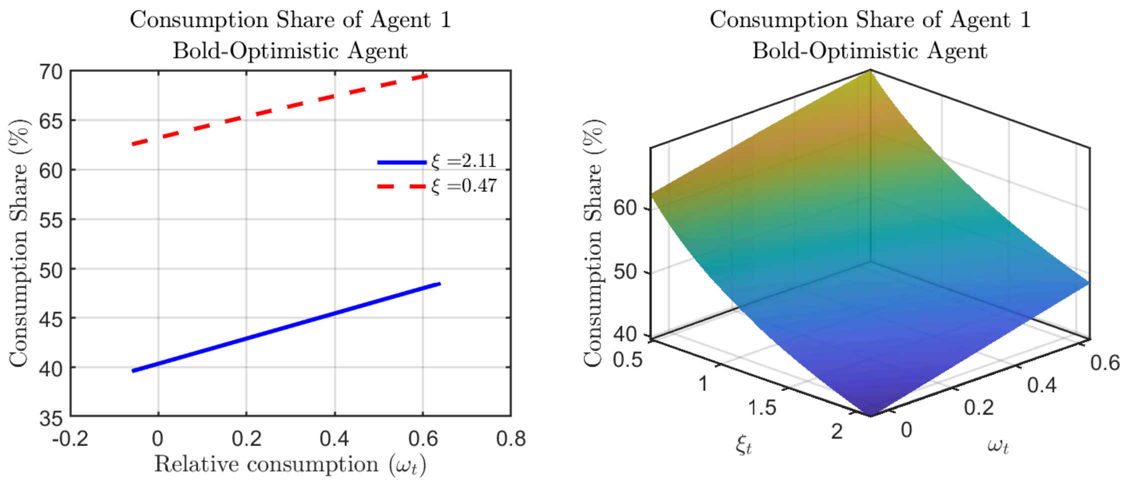

Figure 1: **Optimal Consumption (the Bold-Optimistic agent).** The left panel illustrates the consumption share of the Bold-Optimistic agent as a function of relative (log) consumption, ω_t, for two levels of belief disagreement, ξ_t. The right panel depicts the consumption share of the Bold-Optimistic agent over the grid of both state variables, ω_t and ξ_t. Given that $\omega_t \sim N(\bar{\omega}, \sigma_\omega^2)$ with $\sigma_\omega^2 = \frac{\sigma^2}{2\lambda_x}$, we construct the grid for ω_t over the interval $[\bar{\omega} - 3\sigma_\omega, \bar{\omega} + 3\sigma_\omega]$, which approximately captures 99% of its realizations. Similarly, since $\log \xi_t \sim N(\mu_\xi, \sigma_\xi^2)$, where $\mu_\xi = -\frac{1}{2}(\sigma_{\xi,2}^2 - \sigma_{\xi,1}^2)$ and $\sigma_\xi = \sigma_{\xi,2} - \sigma_{\xi,1}$, we construct the grid for $\log \xi_t$ over the interval $[\mu_\xi - 3\sigma_\xi, \mu_\xi + 3\sigma_\xi]$, also capturing approximately 99% of its realizations. Consequently, the grid for ξ_t is defined as $[\exp(\mu_\xi - 3\sigma_\xi), \exp(\mu_\xi + 3\sigma_\xi)]$. Parameter values correspond to the baseline calibration of our model.

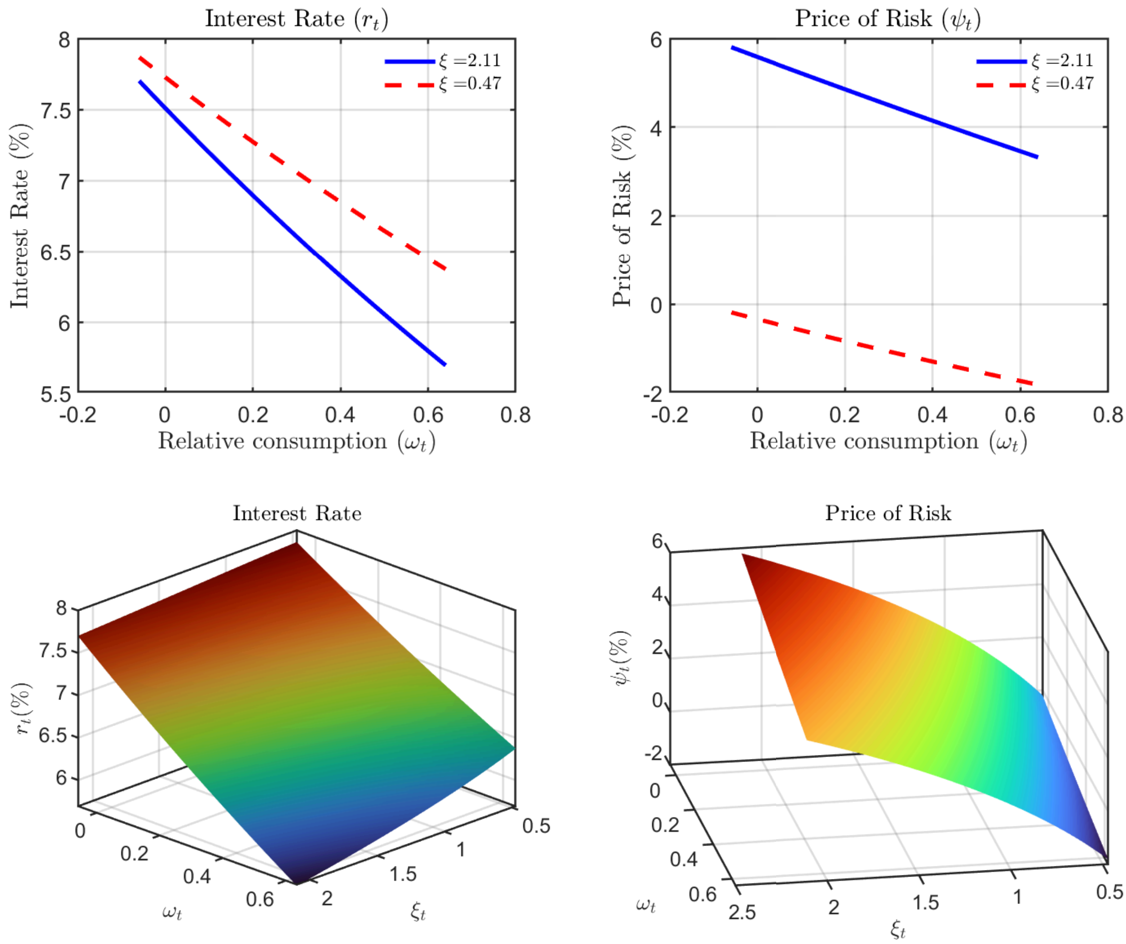

Figure 2: **Equilibrium Interest Rate r_t and Price of Risk ψ_t.** The upper panel displays the interest rate and the price of risk as functions of relative (log) consumption ω for two levels of belief disagreement ξ. The lower panel shows the interest rate and the price of risk over the grid of both state variables, ω and ξ. The parameter values reflect the baseline calibration of our model.

4 The Equity Term Structure

In this section, we study the model's implications for the term structure of equity yields. First, we present facts about the slope and volatility of the equity term structure. Second, we calculate the price of dividend strips based on our model features and then determine the equity term structure. Our goal in this section is to compare the equity term structure generated by our model with the data.

4.1 Preliminary Evidence

We highlight five main characteristics of the term structure of equity yields based on the estimation of Giglio et al. (2024) for 1974-2020. Specifically, we use their estimated equity term structure for the aggregate market index to determine its features. First, its average slope is procyclical, showing a positive slope in normal times and a negative slope in recessions. Second, the level of equity yields is countercyclical across maturities, above the unconditional yields in recessions and below them in normal times. Third, the unconditional average slope exhibits a positive sign.[2] Fourth, the volatility of the term structure of equity yields behaves countercyclically, above the unconditional value in recessions and below it in normal times. Fifth, the volatility of equity yields consistently decreases with maturity, which is consistent with van Binsbergen and Koijen (2017) and van Binsbergen et al. (2012).

4.2 The Price of Dividend Strips

We define a *dividend strip* as an asset that delivers $Y_{t+\tau}$ units of consumption τ periods from now. Its price at time t, denoted by $h_t^{(\tau)}$, is given by:

$$h_{t+n}^{(\tau-n)} = E_{t+n}\left[\frac{m_{t+\tau}}{m_{t+n}} Y_{t+\tau}\right], \quad n \in \{0, 1, \cdots, \tau\}, \tag{29}$$

where $n \in \{0, 1, 2, \cdots, \tau-1, \tau\}$ represents the period for which we calculate the price. By setting $n = 0$ in Eq. (29) and considering the expression for the stochastic discount factor from Eq. (25), the current price of a dividend strip with maturity τ is given by:

$$h_t^{(\tau)} = e^{-\rho\tau} \frac{e^{\gamma_1 \omega_t + x_t}}{\xi_{1,t} \tilde{c}_{1,t}^{-\gamma_1}} E_t \underbrace{\left[e^{-(\gamma_1-1)\omega_{t+\tau}} \xi_{1,t+\tau} \tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}_{f(\omega_{t+\tau}, \xi_{t+\tau}, \xi_{1,t+\tau})}. \tag{30}$$

The fact that the optimal consumption share $\tilde{c}_{k,t}$ for $k \in \{1, 2\}$ is a function of ω_t and ξ_t (as shown in Lemma 1) implies that the expression inside the expectation operator in Eq. (30) depends on $\omega_{t+\tau}$ and $\xi_{t+\tau}$, which are random variables driven by same Brownian shocks. However, this expectation cannot be solved analytically because

[2] Preliminary evidence suggested that the equity term structure has an unconditional downward slope(van Binsbergen et al., 2012; van Binsbergen and Koijen, 2017). However, recent literature shows that this is not the case, the term structure has an unconditional upward slope (Bansal et al., 2021; Giglio et al., 2024; Boguth et al., 2023; Schröder, 2024).

$f(\omega_{t+\tau}, \xi_{t+\tau}, \xi_{1,t+\tau})$ is a nonlinear function of $\omega_{t+\tau}$ and $\xi_{t+\tau}$. Therefore, we solve it using the Gaussian quadrature procedure. The following lemma specifies the price of dividend strips. We leave the details of the implementation of this numerical integration method in Appendix B.

Lemma 2. *The price of dividend strips is given as following, in which a random variable $\tilde{s} \sim \mathcal{N}(0,1)$ is introduced inside the expectation operator.*

$$h_t^{(\tau)} = e^{-\rho\tau} \frac{e^{\gamma_1 \omega_t + x_t - \frac{1}{2}\sigma_{\xi,1}^2 \tau}}{\tilde{c}_{1,t}^{-\gamma_1}} E_t \left[e^{-(\gamma_1 - 1)\omega_{t+\tau}(\tilde{s})} e^{\sigma_{\xi,1}\sqrt{\tau} \times \tilde{s}} \left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1} \right], \quad \tilde{s} \sim \mathcal{N}(0,1), \tag{31}$$

$\omega_{t+\tau}$ and $\xi_{t+\tau}$ are transformed into random variables based on $\tilde{s}$ accordingly, as follows,

$$\omega_{t+\tau} = \bar{\omega} + (\omega_t - \bar{\omega}) e^{-\lambda_x \tau} + \sigma e^{-\lambda_x \tau} \sqrt{\frac{e^{2\lambda_x \tau} - 1}{2\lambda_x}} \times \tilde{s}, \tag{32}$$

$$\xi_{t+\tau} = \xi_t e^{a_\tau + \frac{\mu_2 - \mu_1}{\sigma}\sqrt{\tau} \times \tilde{s}}, \quad \textit{with} \quad a_\tau = -\frac{1}{2}(\sigma_{\xi,2}^2 - \sigma_{\xi,1}^2)\tau. \tag{33}$$

Proof. See Appendix A.2.

This lemma shows that the expression $f(\omega_{t+\tau}, \xi_{t+\tau}, \xi_{1,t+\tau})$ in Eq. (30) can be rewritten as a function of a standard normal variable $\tilde{s}$ and the model's parameters, thereby simplifying the computation of the expectation in the dividend price equation.

4.3 The Term Structure of Equity Yields

We now examine the effects of heterogeneous beliefs on equity yields, defined as:

$$r_t^\tau = \frac{1}{\tau} \log \frac{Y_t}{h_t^{(\tau)}}, \tag{34}$$

where $h_t^{(\tau)}$ denotes the price of a dividend strip with maturity τ, and Y_t is the dividend at time t. The term structure of equity yields refers to the sequence r_t^τ across maturities τ. Our focus is on how differences in beliefs affect the slope of this term structure.

Using the expression for $h_t^{(\tau)}$ from Eq. (31), the following lemma provides a closed-form expression for the equity yield r_t^τ. The numerical evaluation of expectations follows the same procedure outlined in Lemma 2.

Lemma 3. *The equity yield is given as following.*

$$r_t^\tau = \frac{1}{\tau} \log \left[\frac{g(\omega_t, \tilde{c}_{1,t})}{E_t \left[e^{\sigma_{K,\tau}\tilde{s}} \left[\tilde{c}_{1,t+\tau}(s)\right]^{-\gamma_1} \right]} \right]. \tag{35}$$

Proof. See Appendix A.3.

Equation (35) indicates that equity yields depend on the current and future distribution of the state variables. To better understand the determinants of equity yields, we seek an approximate relationship between consumption and the state variable. Although the risk-sharing rule reveals that this relationship is highly nonlinear, it is possible to find a simplified relationship that aids in expressing r_t^τ, as demonstrated in the following Lemma.

Lemma 4. *Given the solution of the theoretical model, the following holds:*

1. ***Optimal Consumption:*** *From risk-sharing rule evaluated at $t+\tau$, using the following approximation: $e^{cc_{1,t}} \approx 1 + cc_{1,t}$, there is an approximate linear relationship between $cc_{1,t+\tau} \equiv \log(\tilde{c}_{1,t+\tau})$ and $\omega_{t+\tau}$ and $\log(\xi_t + \tau)$, given by*

$$cc_{1,t+\tau} \approx -A - B\omega_{t+\tau} - C\log(\xi_{t+\tau}) \tag{36}$$

   where constants A, B, and C are given by

$$A = \frac{\log(\lambda_{2,0}/\lambda_{1,0}) + \gamma_2}{\gamma_1 + \gamma_2}, \qquad B = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}, \qquad C = \frac{1}{\gamma_1 + \gamma_2}. \tag{37}$$

2. ***The Nonlinear Expectation:*** *Using expression (36) evaluated at t, $\log g(\omega_t, \tilde{c}_{1,t})$ can be written as*

$$\begin{aligned}\log g(\omega_t, \tilde{c}_{1,t}) &= \rho\tau - (\gamma_1 - 1)\omega_t + (\gamma_1 - 1)\left(\bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x \tau}\right) + \frac{1}{2}\sigma_{\xi,1}^2\tau - \gamma_1 \log(\tilde{c}_{1,t}) \\ &\approx \rho\tau - (\gamma_1 - 1)\omega_t + (\gamma_1 - 1)\left(\bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x \tau}\right) + \frac{1}{2}\sigma_{\xi,1}^2\tau \\ &+ \gamma_1 A + \gamma_1 B\omega_t + \gamma_1 C\log(\xi_t)\end{aligned} \tag{38}$$

   Furthermore, using expression Eq. (36), the component $\log E_t\left[e^{\sigma_{K,\tau}\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right]$ of equity yields r_t^τ from Eq. (35) can be expressed as follows:

$$\begin{aligned}\log E_t\left[e^{\sigma_{K,\tau}\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right] &\approx \gamma_1 A + \gamma_1 B\left[\bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x\tau}\right] \\ &+ \gamma_1 C\left[\log(\xi_t) + a_\tau\right] + \frac{1}{2}\sigma_{H,\tau}^2,\end{aligned} \tag{39}$$

   where

$$\sigma_{H,\tau} = \sigma_{K,\tau} + \gamma_1 B\sigma e^{-\lambda_x\tau}\sqrt{\frac{e^{2\lambda_x\tau} - 1}{2\lambda_x}} + \frac{\mu_2 - \mu_1}{\sigma}\sqrt{\tau}. \tag{40}$$

3. ***Equity Yields:*** *Using above expressions (38) and (39), the equity yields (Eq. 35) can be expressed as follows:*

$$r_t^\tau \approx \rho + \frac{1}{2}\sigma_{\xi,1}^2 + \frac{1}{\tau}\left[b(\omega_t - \bar{\omega})\left(1 - e^{-\lambda_x\tau}\right) - \gamma_1 C a_\tau - \frac{1}{2}\sigma_{H,\tau}^2\right] \tag{41}$$

   The coefficient b is expressed as $b = (\gamma_1 + \gamma_2 - 2\gamma_1\gamma_2)/(\gamma_1 + \gamma_2) < 0$.

Proof. See Appendix A.4.

This Lemma warrants further discussion. Firstly, there is an approximate linear relationship between (log) consumption and the state variable, primarily driven by preference heterogeneity. Secondly, the expectation component of equity yields is largely explained by the first and second moments of the distribution of $\omega_{t+\tau}$. In this context, habit persistence λ_x plays a crucial role. Thirdly, Equation (41) indicates that equity yields are state-dependent, meaning that in good times ($\omega_t > \bar{\omega}$), r_t^τ is lower than in bad times ($\omega_t < \bar{\omega}$). Additionally, preference heterogeneity, represented by the coefficient b, and habit persistence significantly influence the equity term structure.

In Figure 3,4 we plot the equity yields generated by the heterogeneous-agent model with habits. Interestingly, the model produces an unconditionally downward-sloping term structure of equity yields. Furthermore, it generates a downward-sloping equity term structure in recessions (low values of ω_t) and an upward-sloping one in good times (high values of ω_t). These results are consistent with the findings of Giglio et al. (2024).

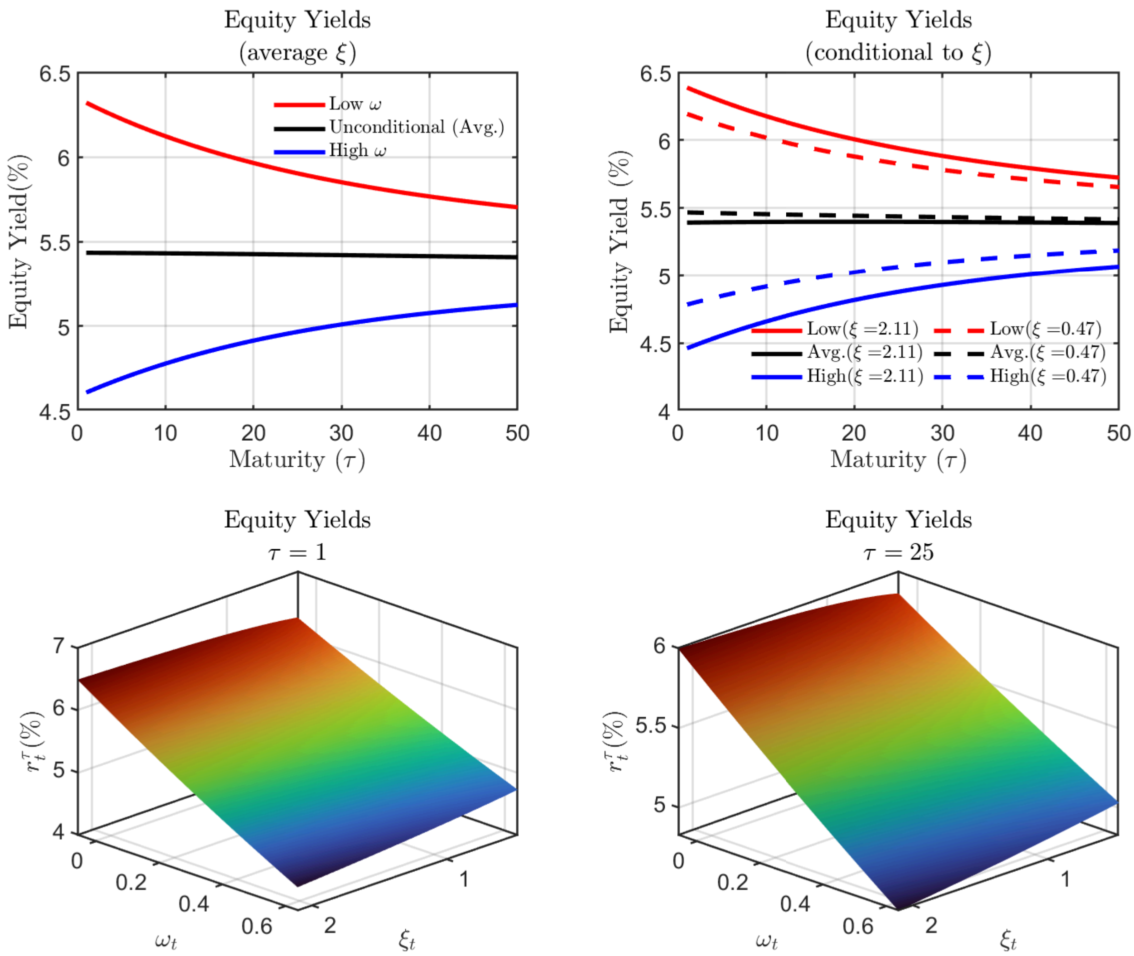

Figure 3: **The Term Structure of Equity Yields (Model Result).** The upper panel illustrates equity yields across maturities τ for recessions (low ω), expansions (high ω), and the unconditional mean. Low ω corresponds to the first decile of ω, while high ω represents the last decile. The lower panel depicts equity yields over the grid of values for ω and ξ at two maturities. The parameter values reflect the baseline calibration of our model.

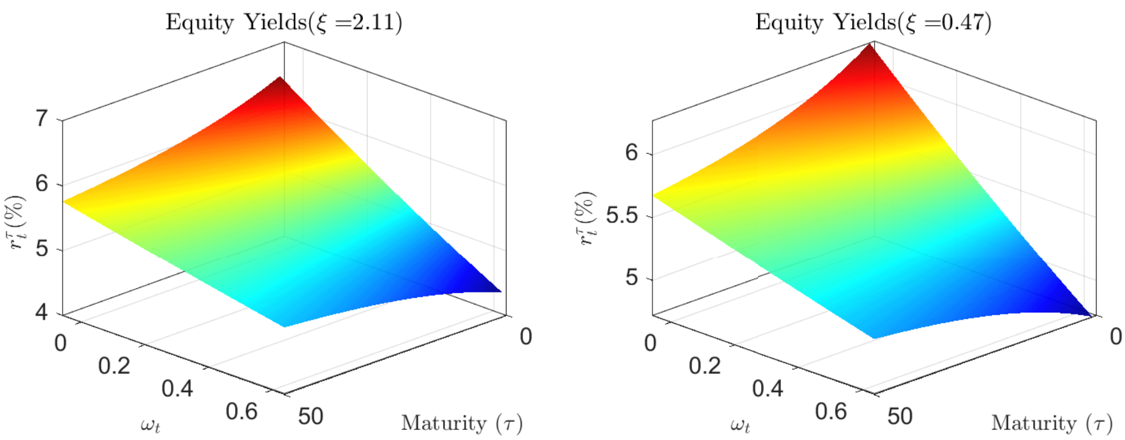

Figure 4: **The Term Structure of Equity Yields (Model Result).** The upper panel illustrates equity yields across maturities τ for recessions (low ω), expansions (high ω), and the unconditional mean. Low ω corresponds to the first decile of ω, while high ω represents the last decile. The lower panel depicts equity yields over the grid of values for ω and ξ at two maturities. The parameter values reflect the baseline calibration of our model.

5 Diagnostic Beliefs

The relevance of diagnostic beliefs. Diagnostic beliefs refer to the tendency of agents to overestimate the probability of a good (bad) future state when current news is good (bad). That is, after evaluating the current state of the economy and forming a diagnostic, agents tend to believe that future states will resemble the present, leading to overreaction (Bordalo et al., 2018, 2020, 2022). As Bordalo et al. (2022) point out, this overreaction is rooted in psychology and has been supported by survey data. Moreover, the literature has shown that diagnostic beliefs play a central role in credit cycles (Bordalo et al., 2018), asset prices (Bordalo et al., 2024), and business cycles (Bigio et al., 2025). Motivated by these studies, we investigate the role of diagnostic beliefs in shaping equity yields.

Modeling diagnostic beliefs. Diagnostic beliefs capture the idea that investors' expectations about fundamentals are heavily influenced by recent news. Since the state variable—relative (log) consumption at time t, denoted by ω_t—reflects recent economic conditions, we model agents' beliefs about the drift of the endowment as follows:

$$\mu_{k,t}(\omega_t) = \mu + \theta_k \cdot f(\omega_t), \quad k \in \{1, 2\}, \tag{42}$$

where $\mu_{k,t}$ denotes agent k's subjective expectation of endowment growth, which is now state dependent. The term $\theta_k f(\omega_t)$ captures the diagnostic distortion from the true drift μ, and the linear function $f(\omega_t) = \omega_t - \bar{\omega}$ characterizes the state-dependent component of diagnostic beliefs. The parameter θ_k represents the strength of agent k's overreaction, and we assume heterogeneity in this dimension:

$$\theta_1 > \theta_2 > 0. \tag{43}$$

Under this specification, (short-term) beliefs are procyclical, consistent with earlier evidence from Greenwood and Shleifer (2014) and more recently from Sias et al. (2024). In good states of the world ($\omega_t > \bar{\omega}$), both agents are optimistic, but agent 1 overreacts more strongly. Conversely, in bad states ($\omega_t < \bar{\omega}$), both agents are pessimistic, with agent 1 again displaying a stronger overreaction. We illustrate these beliefs in Figure 5.

Belief disagreement. To characterize belief disagreement between agents, we first recall Girsanov's theorem, which links the Brownian motion under agent k's subjective measure $\mathbb{P}^k$ to the objective measure $\mathbb{P}$:

$$dZ_t = d\tilde{Z}_t^k + \sigma_{\tilde{\xi},k}(\omega_t)\, dt, \quad \text{where} \quad \sigma_{\tilde{\xi},k}(\omega_t) = \frac{\mu_{k,t}(\omega_t) - \mu}{\sigma} \equiv \frac{\theta_k}{\sigma}(\omega_t - \bar{\omega}). \tag{44}$$

As discussed in Section 2.3, $\sigma_{\tilde{\xi},k}(\omega_t)$ represents the extent of agent k's belief distortion per unit of risk. Unlike the constant-belief case (i.e., $\mu_{k,t}(\omega) = \mu_k$), this disagreement is now state dependent and characterized by an overreaction component. Consequently, the Radon-Nikodym derivative of agent k's subjective measure $\mathbb{P}^k$ with respect to the objective measure $\mathbb{P}$ takes the form:

$$\tilde{\xi}_{k,t} = \exp\left(-\frac{1}{2}\int_0^t \sigma_{\tilde{\xi},k}^2(\omega_s)\, ds + \int_0^t \sigma_{\tilde{\xi},k}(\omega_s)\, dZ_s\right), \tag{45}$$

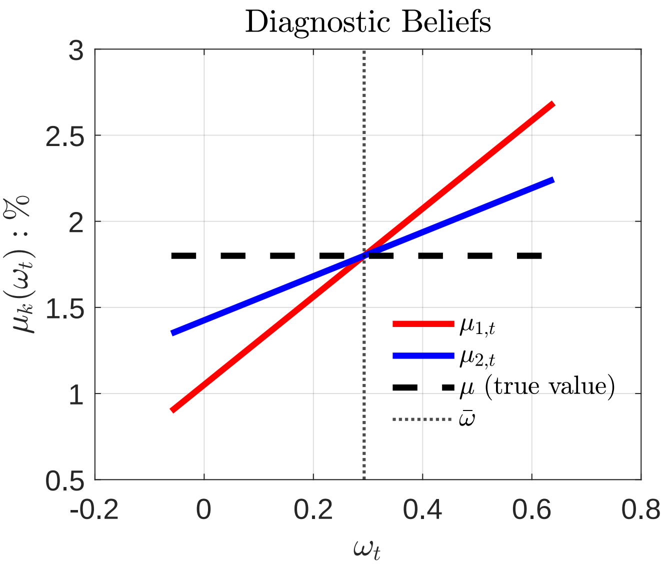

Figure 5: **Diagnostic Beliefs.** This figure plots $\mu_{k,t} = \mu + \theta_k \cdot (\omega_t - \bar{\omega})$ for both agents, $k \in \{1, 2\}$, and compares them with the true value of the endowment growth rate, μ. We set $\theta_1 = 0.5\theta^{\max}$ and $\theta_2 = 0.25\theta^{\max}$ with $\theta^{\max} = \mu/3\sigma_f$. The parameter values used are based on Table 1.

with dynamics defined by:

$$\frac{d\tilde{\xi}_{k,t}}{\tilde{\xi}_{k,t}} = \sigma_{\tilde{\xi},k} dZ_t \tag{46}$$

We define the aggregate level of belief disagreement, under diagnostic beliefs, as:

$$\tilde{\xi}_t \equiv \frac{\tilde{\xi}_{2,t}}{\tilde{\xi}_{1,t}} = \exp\left(-\frac{1}{2}\int_0^t \left[\sigma^2_{\tilde{\xi},2}(\omega_s) - \sigma^2_{\tilde{\xi},1}(\omega_s)\right] ds + \int_0^t \sigma_{\tilde{\xi}}(\omega_s)\, dZ_s\right), \tag{47}$$

where the belief difference function $\sigma_{\tilde{\xi}}(\omega_t)$ is defined as:

$$\sigma_{\tilde{\xi}}(\omega_t) = \sigma_{\tilde{\xi},2}(\omega_t) - \sigma_{\tilde{\xi},1}(\omega_t) = \frac{\mu_{2,t}(\omega_t) - \mu_{1,t}(\omega_t)}{\sigma} \equiv \frac{\theta_2 - \theta_1}{\sigma}(\omega_t - \bar{\omega}). \tag{48}$$

To fully characterize $\tilde{\xi}_t$, we apply Itô's lemma to Eq. (47) to derive its dynamics:

$$\frac{d\tilde{\xi}_t}{\tilde{\xi}_t} = -\sigma_{\tilde{\xi},1}\sigma_{\tilde{\xi}} dt + \sigma_{\tilde{\xi}} dZ_t, \tag{49}$$

Optimality and equilibrium. The agent's optimal control problem described in Eq. (19), as well as the definition of equilibrium outlined in Section 2.4, remain unchanged in structure. Consequently, the risk-sharing rule in Eq. (24) and the stochastic discount factor in Eq. (25) also retain their form but now incorporate the updated definition of the Radon-Nikodym derivative, denoted by $\tilde{\xi}_t$ and given in Eq. (45). Equipped with these objects, we proceed to examine the role of diagnostic beliefs in shaping equity yields.

Discussion on the values of θ_k. To determine a plausible range for θ_k, we use the fact that $f(\omega_t) = \omega_t - \bar{\omega} \sim \mathcal{N}(0, \sigma_f^2)$, where $\sigma_f = \sqrt{\sigma^2/(2\lambda_x)}$, along with the constraint that diagnostic beliefs must remain positive, i.e., $\mu_{k,t}(\omega_t) > 0$.

We focus on the interval $[-3\sigma_f, 3\sigma_f]$, which captures approximately 99% of the distribution of $f(\omega_t)$. Within this range, the distorted belief is given by $\mu_{k,t} = \mu + \theta_k f(\omega_t)$, implying:

$$\mu - 3\theta_k\sigma_f \leq \mu_{k,t} \leq \mu + 3\theta_k\sigma_f.$$

To ensure positivity of $\mu_{k,t}$ across this range, we impose:

$$0 < \mu - 3\theta_k\sigma_f \leq \mu_{k,t} \leq \mu + 3\theta_k\sigma_f.$$

This condition yields an upper bound for θ_k:

$$\theta_k < \frac{\mu}{3\sigma_f}.$$

Since $\theta_k = 0$ corresponds to the benchmark case of homogeneous beliefs (i.e., $\mu_{1,t} = \mu_{2,t} = \mu$), a strictly positive θ_k is required to introduce belief heterogeneity. Therefore, the feasible range is:

$$0 < \theta_k < \theta^{\max}, \quad \text{where} \quad \theta^{\max} = \frac{\mu}{3\sigma_f}.$$

Given this bound, we have flexibility in selecting specific values for θ_k. In our baseline scenario, we set $\theta_1 = 0.5\theta^{\max}$ and $\theta_2 = 0.25\theta^{\max}$, implying that agent 1 overreacts twice as strongly as agent 2 (i.e., $\theta_1 = 2\theta_2$).

5.1 The Term Structure of Equity Yields

We first rewrite the dividend price, Eq. (30), taking account the Radon-Nikodym derivative $\tilde{\xi}_{1,t}$ with diagnostic beliefs, as follows:

$$h_t^{(\tau)} = e^{-\rho\tau} \frac{e^{\gamma_1\omega_t + x_t}}{\tilde{\xi}_{1,t}\tilde{c}_{1,t}^{-\gamma_1}} E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}}\tilde{\xi}_{1,t+\tau}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]. \tag{50}$$

Recall the definition of equity yields from Eq. (34):

$$r_t^\tau = \frac{1}{\tau}\log\frac{Y_t}{h_t^{(\tau)}}. \tag{51}$$

Considering the expression for dividend price, $h_t^{(\tau)}$, defined by Eq. (50), equity yields can be expressed as:

Lemma 5. *Equity yields can be expressed as:*

$$r_t^\tau = \rho - \frac{1}{\tau}\log E_t\left[e^{\alpha}\right], \tag{52}$$

where function α *is defined as:*

$$\alpha = a(\log\tilde{\xi}_{t+\tau} - \log\tilde{\xi}_t) + b(\omega_{t+\tau} - \omega_t) + (\log\tilde{\xi}_{1,t+\tau} - \log\tilde{\xi}_{1,t}), \tag{53}$$

where $a = \gamma_1/(\gamma_1+\gamma_2)$ *and* $b = (\gamma_1+\gamma_2-2\gamma_1\gamma_2)/(\gamma_1+\gamma_2) < 0$.

Proof. See Appendix A.5.

Equation (52) indicates that equity yields depend on three state variables: $\tilde{\xi}_s$, $\tilde{\xi}_{1,s}$, and ω_s for $s \in [t, t+\tau]$. In particular, equity yields are determined by the expected values of these variables. Since diagnostic beliefs are state-dependent and influenced by the current economic condition ω_t, these variables are interrelated. This dependence simplifies the computation of expectations in the equity yield expression (52), as shown in the following lemma.

Lemma 6. ***Function*** α**.** *The function α depends on the maturity τ and on the path of the process $\omega_s - \bar{\omega}$ accumulated over the interval $s \in [t, t+\tau]$. Specifically, α can be expressed as:*

$$\alpha = \alpha\left(\tau, \{\omega_s - \bar{\omega}\}_t^{t+\tau}, \{dZ_s\}_t^{t+\tau}; \Theta\right), \tag{54}$$

where Θ denotes the set of model parameters, including those governing risk aversion heterogeneity and heterogeneous diagnostic beliefs.

Explicitly, $\alpha(\cdot)$ is given by:

$$\alpha = \alpha_0 \int_t^{t+\tau} (\omega_s - \bar{\omega})^2 \, ds + \alpha_1 \int_t^{t+\tau} (\omega_s - \bar{\omega}) \, dZ_s - b\lambda_x \int_t^{t+\tau} (\omega_s - \bar{\omega}) \, ds + b\sigma \int_t^{t+\tau} dZ_s, \tag{55}$$

where:

$$\alpha_0 = -\frac{(1-a)\theta_1^2 + a\theta_2^2}{2\sigma^2}, \qquad \alpha_1 = \frac{(1-a)\theta_1 + a\theta_2}{\sigma},$$

and the parameters a and b are defined as:

$$a = \frac{\gamma_1}{\gamma_1 + \gamma_2}, \qquad b = \frac{\gamma_1 + \gamma_2 - 2\gamma_1\gamma_2}{\gamma_1 + \gamma_2} < 0,$$

with θ_1 and θ_2 denoting the overreaction parameters for each agent.

Proof. See Appendix A.6.

This lemma indicates that the function α depends on the sample paths of the deviation of relative (log) consumption from its long-run mean, as well as on the path of shocks over the interval $[t, t+\tau]$. This formulation relates equity yields to the state variable ω_t and its evolution over time until maturity. Although this approach avoids the explicit use of $\tilde{\xi}_t$ and $\tilde{\xi}_{1,t}$ in the computation of equity yields, the calculation of the expectation $E_t[e^\alpha]$ in Eq. (52) remains complex and requires numerical integration.

Specifically, we employ Monte Carlo simulation, taking advantage of the fact that ω_s follows an Ornstein–Uhlenbeck process. For each maturity τ and a fixed initial state ω_t, we discretize the path $\{\omega_s\}_t^{t+\tau}$ using $N = 5000$ time steps. We then approximate the deterministic and stochastic integrals in Eq. (55) with discrete summations to compute α, and hence $\exp(\alpha)$. We perform $M = 50,000$ simulations to approximate the expectation $E_t[e^\alpha]$ as the average of $\exp(\alpha)$ across the M simulated paths. The complete procedure is detailed in Appendix C.

Based on our numerical results for $E_t[e^{\alpha}]$, we proceed to compute equity yields using Eq. (52), and present the results in Figure 6. Panel A compares equity yields under two scenarios: one with homogeneous beliefs equal to the true value ($\mu_1 = \mu_2 = \mu$), achieved by setting $\theta_k = 0$, and another where agents hold diagnostic beliefs.

In expansions, the less risk-averse agent overreacts to good news, expecting future states to resemble the favorable current conditions. Anticipating high cash flows from risky assets, he reduces his exposure to those assets—such as dividend strips—leading to lower asset prices and, consequently, higher equity yields compared to the homogeneous-belief case.

In recessions, this agent similarly overreacts, believing that poor economic conditions will persist. However, his higher risk tolerance partially offsets his pessimistic beliefs, prompting a reallocation toward risky assets, which in turn reduces equity yields. This offsetting mechanism depends on the parameter values, as demonstrated in Panel B.

Panel B illustrates the effect of increasing the overreaction parameter for the less risk-averse agent (θ_1). In expansions, stronger overreaction further reduces his demand for risky assets, pushing equity yields even higher. In contrast, during recessions, more pessimistic beliefs dominate the agent's risk tolerance, requiring higher equity yields to hold dividend strips.

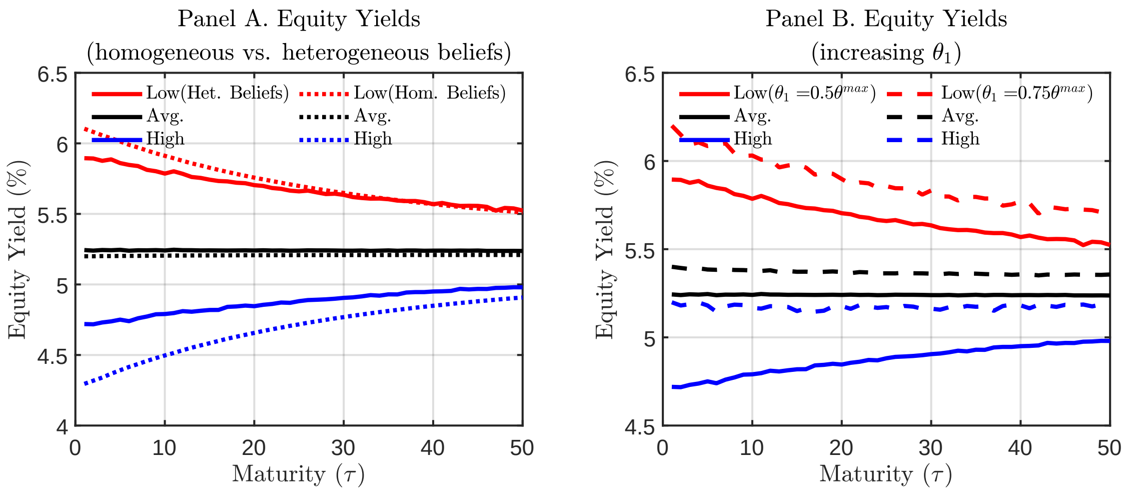

Figure 6: **The Term Structure of Equity Yields (Diagnostic Beliefs).** Panel A illustrates equity yields across maturities τ for recessions (low ω), expansions (high ω), and the unconditional mean. Low values of ω correspond to the first decile of its distribution, while high values represent the last decile. Panel B displays equity yields for both the baseline case (as shown in Panel A) and a scenario in which θ_1 increases from $0.5\theta^{max}$ to $0.75\theta^{max}$, capturing a stronger overreaction by agent 1. The parameter θ^{max} is set up at $\mu/(3\sqrt{\sigma^2/(2\lambda_x)})$. Given that $\omega_t \sim N(\bar{\omega}, \sigma_\omega^2)$ with $\sigma_\omega^2 = \frac{\sigma^2}{2\lambda_x}$, we construct the grid for ω_t over the interval $[\bar{\omega} - 3\sigma_\omega, \bar{\omega} + 3\sigma_\omega]$, which approximately covers 99% of its realizations. The model's parameter values are based on Table 1.

6 Conclusions

This paper studies how heterogeneity in belief and risk aversion affect the slope of the equity term structure. Our endowment economy is populated by two agents with different risk aversion and beliefs, and "catching up with the Joneses" preferences. The two agents trade in dynamically complete markets. The habit preference allows the model to remain stationary, ensuring the survival of both agents in the long run.

We show that the model is able to produce a procyclical slope of equity yields, consistent with empirical evidence (van Binsbergen et al., 2013; Bansal et al., 2021; Giglio et al., 2024). Next, we show that belief heterogeneity has significant implications for equity yields. First, the impact of belief heterogeneity depends on the business cycle. In recessions, it raises equity yields across maturities, while in expansions it reduces them. Second, the level effect on equity yields is more pronounced during recessions than expansions. Third, the magnitude of the effect is stronger for short-term assets than for long-term assets.

Our results open several interesting research avenues. First, the model can be improved by incorporating state-dependent variance of the state variable, which would allow it to capture the conditional variance of equity yields—a feature not currently captured by our model. Second, similar to the term structure of interest rates, the elasticity of intertemporal substitution plays a key role in determining its shape. Therefore, exploring whether this elasticity influences the equity term structure would be informative. To this end, the model could be extended to consider recursive preferences. We leave these extensions for future work.

Appendix

A Lemma Proofs

A.1 Proof of Lemma 1.

Optimal consumption. The social planner solves the following static optimization problem P in period t:

$$\sup_{\{c_{1,t},c_{2,t}\}} \left\{e^{-\rho t}\left[\lambda_{1,t}u(c_{1,t},X_t)+\lambda_{2,t}u(c_{2,t},X_t)\right]\right\} \tag{56}$$

subject to

$$c_{1,t}+c_{2,t}\leqslant Y_t \tag{57}$$

The Lagrange function associated with P is

$$\mathcal{L}=\left\{e^{-\rho t}\left[\lambda_{1,t}u(c_{1,t},X_t)+\lambda_{2,t}u(Y_t-c_{1,t},X_t)\right]\right\} \tag{58}$$

The FOC is given by

$$c_{1,t}:\frac{\partial\mathcal{L}}{\partial c_{1,t}}=e^{-\rho t}\lambda_{1,t}u_{c_{1,t}}+e^{-\rho t}\lambda_{2,t}u_{c_{2,t}}(-1)=0 \tag{59}$$

Given Eq. (9), $\lambda_{k,t}\equiv\lambda_{k,0}\xi_{k,t}$ is the stochastic weight of the agent-k's utility in the social planner utility function, and $\lambda_{k,0}$ is the initial endowment of agent-k, Eq. (59) becomes

$$\lambda_{1,0}\xi_{1,t}e^{-\rho t}(\frac{1}{X_t})^{1-\gamma_1}c_{1,t}^{-\gamma_1}=\lambda_{2,0}\xi_{2,t}e^{-\rho t}(\frac{1}{X_t})^{1-\gamma_2}c_{2,t}^{-\gamma_2} \tag{60}$$

The marginal utility of the social planner is the Stochastic Discount Factor (SDF). Defining m_t as the equilibrium SDF, and $\tilde{c}_{k,t}\equiv\frac{c_{k,t}}{Y_t}$ as consumption share, the above expression can be rewritten as

$$m_t\equiv\hat{m}_{1,t}\tilde{c}_{1,t}^{-\gamma_1}=\hat{m}_{2,t}\tilde{c}_{2,t}^{-\gamma_2}, \tag{61}$$

where $\hat{m}_{k,t}$, defined below, is the SDF under the physical probability measure $\mathcal{P}$ when agent-k is the sole agent in the economy:

$$\hat{m}_{k,t}=\lambda_{k,0}\xi_{k,t}e^{-\rho t}e^{-\gamma_k\omega_t-x_t}. \tag{62}$$

The Interest rate r_t and the price of risk ψ_t. For the interest rate dynamics and price of risk, we have

$$\frac{dm_t}{m_t}=-r_tdt-\psi_tdZ_t \tag{63}$$

So we need to apply Itô's lemma to m_t, given

$$dx_t=\lambda_x\omega_tdt \tag{64}$$

$$d\omega_t=\lambda_x(\bar{\omega}-\omega_t)dt+\sigma dZ_t \tag{65}$$

$$\frac{d\xi_{k,t}}{\xi_{k,t}} = \sigma_{\xi,k} dZ_t \tag{66}$$

We also define the dynamics of $\hat{m}_{1,t}$ and $\tilde{c}_{1,t}$ as follows:

$$\frac{d\hat{m}_{1,t}}{\hat{m}_{1,t}} = -\hat{r}_{1t}dt - \hat{\psi}_{1,t}dZ_t \tag{67}$$

$$\frac{d\tilde{c}_{1,t}}{\tilde{c}_{1,t}} = \mu_{\tilde{c}_{1},t}dt + \sigma_{\tilde{c}_{1},t}dZ_t \tag{68}$$

Applying Itô's lemma to equations (67) and (68), we find the dynamic of their "log" as follows:

$$d\ln \hat{m}_{1,t} = \left(-\hat{r}_{1,t} - \frac{1}{2}\hat{\psi}_{1,t}^2\right)dt - \hat{\psi}_{1,t}dZ_t \tag{69}$$

$$d\ln \tilde{c}_{1,t} = \left(\mu_{\tilde{c}_1,t} - \frac{1}{2}\sigma_{\tilde{c}_1,t}^2\right)dt + \sigma_{\tilde{c}_1,t}dZ_t \tag{70}$$

We use the SDF based on agent 1, then we take "log" on equation (61) as follows:

$$\ln m_t = \ln \hat{m}_{1,t} - \gamma_1 \ln \tilde{c}_{1,t} \tag{71}$$

By Itô's lemma:

$$d\ln m_t = d\ln \hat{m}_{1,t} - \gamma_1 d\ln \tilde{c}_{1,t} \tag{72}$$

$$d\ln m_t = -\left[\left(\hat{r}_{1,t} + \frac{1}{2}\hat{\psi}_{1,t}^2\right) + \gamma_1\left(\mu_{\tilde{c}_1,t} - \frac{1}{2}\sigma_{\tilde{c}_1,t}^2\right)\right]dt - \left[\hat{\psi}_{1t} + \gamma_1\sigma_{\tilde{c}_1,t}\right]dZ_t \tag{73}$$

To fully identify equation (73), we need to find the drift and diffusion terms of both $d\ln \hat{m}_{1,t}$ and $d\ln \tilde{c}_{1,t}$.

From the definition (62), we know:

$$\hat{m}_{1,t} = e^{-\rho t}\lambda_{1,0}\xi_{1,t}e^{-\gamma_1\omega_t - x_t} \tag{74}$$

Applying "log", we have:

$$\ln \hat{m}_{1,t} = -\rho t + \ln \lambda_{1,0} + \ln \xi_{1,t} - \gamma_1\omega_t - x_t \equiv f(t, \xi_{1,t}, \omega_t, x_t) \tag{75}$$

So we need to apply Itô's lemma to $\ln \hat{m}_{1,t}$, given

$$dx_t = \lambda_x \omega_t dt \tag{76}$$

$$d\omega_t = \lambda_x(\bar{\omega} - \omega_t)dt + \sigma dZ_t \tag{77}$$

$$d\ln \xi_{1t} = -\frac{1}{2}\sigma_{\xi,1}^2 + \sigma_{\xi,1}dZ_t \tag{78}$$

where, equation (78) is derived by applying Itô's lemma to (**??**). Since the terms in equation (75) are presented in summation form, the following holds:

$$d\ln\hat{m}_{1,t} = -\rho dt + \underbrace{d\ln\lambda_{1,0}}_{=0} + d\ln\xi_{1,t} - \gamma_1 d\omega_t - dx_t \tag{79}$$

Then, using Eq. (76)-(78) into (79), we have:

$$d\ln\hat{m}_{1,t} = -\rho dt - \frac{1}{2}\sigma_{\xi,1}^2 dt + \sigma_{\xi,1}dZ_t - \gamma_1[\lambda_x(\bar{\omega}-\omega_t)dt + \sigma dZ_t] - \lambda_x\omega_t dt \tag{80}$$

Rearranging terms:

$$d\ln\hat{m}_{1,t} = \left(-\rho - \frac{1}{2}\sigma_{\xi,1}^2 - \gamma_1\lambda_x(\bar{\omega}-\omega_t) - \lambda_x\omega_t\right)dt + (\sigma_{\xi,1} - \gamma_1\sigma)\,dZ_t \tag{81}$$

Next, we compare Eq. (81) with Eq. (69):

$$d\ln\hat{m}_{1,t} = \left(-\hat{r}_{1,t} - \frac{1}{2}\hat{\psi}_{1,t}^2\right)dt - \hat{\psi}_{1,t}dZ_t \tag{82}$$

$$\hat{\psi}_{1,t} = \gamma_1\sigma - \sigma_{\xi,1} = \gamma_1\sigma - \frac{\mu_1-\mu}{\sigma} \tag{83}$$

where $\sigma_{\xi,1} = \frac{\mu_1-\mu}{\sigma}$.

$$\hat{r}_{1,t} + \frac{1}{2}\hat{\psi}_{1,t}^2 = \rho + \frac{1}{2}\sigma_{\xi,1}^2 + \gamma_1\lambda_x(\bar{\omega}-\omega_t) + \lambda_x\omega_t \tag{84}$$

$$\hat{r}_{1,t} = \rho + \frac{1}{2}\sigma_{\xi,1}^2 + \gamma_1\lambda_x(\bar{\omega}-\omega_t) + \lambda_x\omega_t - \frac{1}{2}\hat{\psi}_{1,t}^2 \tag{85}$$

$$\hat{r}_{1,t} = \rho + \mu_1 + \lambda_x(\gamma_1 - 1)(\bar{\omega}_1 - \omega_t) - \frac{\sigma^2}{2}\left(\gamma_1^2 + 1\right) \tag{86}$$

$$\hat{r}_{1,t} = \rho + \gamma_1\mu_1 - \lambda_x(\gamma_1-1)\,\omega_t - \frac{1}{2}\sigma^2\gamma_1(1+\gamma_1)\,, \tag{87}$$

Now we need the dynamics of the **consumption share** $\tilde{c}_{1,t}$. We can observe that $\tilde{c}_{1,t}$ is a function of $\{\omega_t, \xi_t\}$, from the consumption-sharing rule

$$\tilde{c}_{1,t}^{-\gamma_1} = \frac{\lambda_{2,0}}{\lambda_{1,0}}\xi_t e^{(\gamma_1-\gamma_2)\omega_t}(1-\tilde{c}_{1,t})^{-\gamma_2} \longrightarrow \tilde{c}_{1,t} = f(\omega_t, \xi_t) \tag{88}$$

considering:

$$d\omega_t = \lambda_x(\bar{\omega}-\omega_t)dt + \sigma dZ_t \tag{89}$$

$$\frac{d\xi_t}{\xi_t} = -\sigma_{\xi,1}(\sigma_{\xi,2} - \sigma_{\xi,1})dt + \frac{\mu_2-\mu_1}{\sigma}dZ_t \tag{90}$$

We need implicit differentiation to take derivatives of $\tilde{c}_{1,t}$ w.r.t $\{\omega_t, \xi_t\}$. Then from Itô's lemma, we have the dynamics for $\tilde{c}_{1,t}$. Bring Eq. (68) here:

$$\frac{d\tilde{c}_{1,t}}{\tilde{c}_{1,t}} = \mu_{\tilde{c}_{1,t}}dt + \sigma_{\tilde{c}_{1,t}}dZ_t \tag{91}$$

with derivatives obtained from the risk-sharing rule:

$$\frac{\partial f}{\partial \omega_t} = -(\gamma_1-\gamma_2)\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t \tag{92}$$

$$\frac{\partial f}{\partial \xi_t} = -\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\frac{\mathbf{R}_t}{\xi_t} \tag{93}$$

$$\frac{\partial^2 f}{\partial \omega_t \partial \xi_t} = (\gamma_1-\gamma_2)\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1^3\gamma_2^3}\frac{\mathbf{R}_t^3}{\xi_t}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right) \tag{94}$$

$$\frac{\partial^2 f}{\partial \omega_t^2} = (\gamma_1-\gamma_2)^2\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}\mathbf{R}_t^3}{\gamma_1^3\gamma_2^3}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right) \tag{95}$$

$$\frac{\partial^2 f}{\partial \xi_t^2} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2\xi_t^2}\left(\frac{\mathbf{R}_t}{\gamma_1\gamma_2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)+\frac{\gamma_1\gamma_2}{\mathbf{R}_t}\right) \tag{96}$$

A. Drift term:

$$\begin{aligned}\mu_{\tilde{c}_{1,t}\tilde{c}_{1,t}} = \lambda_x(\bar{\omega}-\omega_t)\frac{\partial f}{\partial \omega_t} \\ -\sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1})\xi_t\frac{\partial f}{\partial \xi_t} \\ +\frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial \omega_t^2} \\ +\frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\xi_t^2\frac{\partial^2 f}{\partial \xi_t^2} \\ +(\mu_2-\mu_1)\,\xi_t\frac{\partial f}{\partial \omega_t\partial \xi_t}\end{aligned} \tag{97}$$

Introducing the derivatives:

$$\begin{aligned}\mu_{\tilde{c}_{1,t}\tilde{c}_{1,t}} = \lambda_x(\bar{\omega}-\omega_t)\left(-(\gamma_1-\gamma_2)\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\right) \\ -\sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1})\xi_t\left(-\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\frac{\mathbf{R}_t}{\xi_t}\right) \\ +\frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}\mathbf{R}_t^3}{\gamma_1^3\gamma_2^3}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\ +\frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\xi_t^2\left(\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2\xi_t^2}\left(\frac{\mathbf{R}_t}{\gamma_1\gamma_2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)+\frac{\gamma_1\gamma_2}{\mathbf{R}_t}\right)\right) \\ +(\mu_2-\mu_1)\,\xi_t\left((\gamma_1-\gamma_2)\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1^3\gamma_2^3}\frac{\mathbf{R}_t^3}{\xi_t}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\end{aligned} \tag{98}$$

Then, factorizing a common term:

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}\tilde{c}_{1,t}} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t \Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1}) \\
& + \frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\left(\frac{\mathbf{R}_t}{\gamma_1\gamma_2}\left(\frac{\mathbf{R}_t}{\gamma_1\gamma_2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)+\frac{\gamma_1\gamma_2}{\mathbf{R}_t}\right)\right) \\
& + (\mu_2-\mu_1)\left((\gamma_1-\gamma_2)\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\Bigg\}
\end{aligned} \tag{99}
$$

Next, working on the fourth line:

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}\tilde{c}_{1,t}} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t \Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1}) \\
& + \frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\left(\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)+1\right) \\
& + (\mu_2-\mu_1)\left((\gamma_1-\gamma_2)\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\Bigg\}
\end{aligned} \tag{100}
$$

Then, split the forth line:

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}\tilde{c}_{1,t}} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t \Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1}) \\
& + \frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\left(\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)+\frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2 \\
& + (\mu_2-\mu_1)\left((\gamma_1-\gamma_2)\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\Bigg\}
\end{aligned} \tag{101}
$$

Moving the last term of the fourth line to the second line

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}}\tilde{c}_{1,t} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \sigma_{\xi,1}(\sigma_{\xi,2}-\sigma_{\xi,1}) + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2 \\
& + \frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\left(\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + (\mu_2-\mu_1)\left((\gamma_1-\gamma_2)\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\Bigg\}
\end{aligned} \tag{102}
$$

Using the definition of $\sigma_{\xi,1}$ and doing some algebra, the second line becomes

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}}\tilde{c}_{1,t} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \frac{(\mu_2-\mu_1)}{\sigma^2}\left(\frac{\mu_1+\mu_2}{2}-\mu\right) \\
& + \frac{1}{2}\sigma^2\left((\gamma_1-\gamma_2)^2\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + \frac{1}{2}\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2\left(\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right) \\
& + (\mu_2-\mu_1)\left((\gamma_1-\gamma_2)\frac{\mathbf{R}_t^2}{\gamma_1^2\gamma_2^2}\left(\gamma_1\tilde{c}_{2,t}^2-\gamma_2\tilde{c}_{1,t}^2\right)\right)\Bigg\}
\end{aligned} \tag{103}
$$

After doing algebra in the last three lines and using the identity $\tilde{c}_{k,t} = \gamma_k w_{k,t}/\mathbf{R}_t$, they are summarized in a single expression:

$$
\begin{aligned}
\mu_{\tilde{c}_{1,t}}\tilde{c}_{1,t} = \frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\Bigg\{ & -(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \\
& + \frac{(\mu_2-\mu_1)}{\sigma^2}\left(\frac{\mu_1+\mu_2}{2}-\mu\right) \\
& + \frac{1}{2}\frac{\gamma_2 w_{2,t}^2-\gamma_1 w_{1,t}^2}{\gamma_1\gamma_2}\left[\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2 + 2(\mu_2-\mu_1)(\gamma_1-\gamma_2) + \sigma^2(\gamma_1-\gamma_2)^2\right]\Bigg\}
\end{aligned} \tag{104}
$$

Dividing by $\tilde{c}_{1,t}$:

$$\mu_{\tilde{c}_1,t} = \frac{\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\Big\{-(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t)$$
$$+\frac{(\mu_2-\mu_1)}{\sigma^2}\left(\frac{\mu_1+\mu_2}{2}-\mu\right)$$
$$+\frac{1}{2}\frac{\gamma_2 w_{2,t}^2-\gamma_1 w_{1,t}^2}{\gamma_1\gamma_2}\left[\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2+2(\mu_2-\mu_1)(\gamma_1-\gamma_2)+\sigma^2(\gamma_1-\gamma_2)^2\right]\Big\} \quad (105)$$

B. Diffusion term:

$$\sigma_{\tilde{c}_1,t}\tilde{c}_{1,t} = \sigma\frac{\partial f}{\partial\omega_t}+\xi_t\frac{\mu_2-\mu_1}{\sigma}\frac{\partial f}{\partial\xi_t} \quad (106)$$
$$=\sigma\left(-(\gamma_1-\gamma_2)\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\right)+\xi_t\frac{\mu_2-\mu_1}{\sigma}\left(-\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\frac{\mathbf{R}_t}{\xi_t}\right)$$
$$=\frac{\tilde{c}_{1,t}\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\left(-\sigma(\gamma_1-\gamma_2)-\frac{\mu_2-\mu_1}{\sigma}\right)$$
$$\sigma_{\tilde{c}_1,t} = \frac{\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t\left(\sigma(\gamma_2-\gamma_1)+\sigma_{\xi,1}-\sigma_{\xi,2}\right) \quad (107)$$

where $\sigma_{\xi,i}=(\mu_i-\mu)/\sigma$ for $i=1,2$.

Bring Eq. (73) here,

$$d\ln m_t = -\left[\left(\hat{r}_{1,t}+\frac{1}{2}\hat{\psi}_{1,t}^2\right)+\gamma_1\left(\mu_{\tilde{c}_1,t}-\frac{1}{2}\sigma_{\tilde{c}_1,t}^2\right)\right]dt-\left[\hat{\psi}_{1,t}+\gamma_1\sigma_{\tilde{c}_1,t}\right]dZ_t \quad (108)$$

Apply Itô's lemma to Eq. (63), we have

$$d\ln m_t = \left(-r_t-\frac{1}{2}\psi_t^2\right)dt-\psi_t dZ_t \quad (109)$$

We next compare Eq. (109) with Eq. (108).

A. Diffusion Term: Price of risk

$$\psi_t = \hat{\psi}_{1,t}+\gamma_1\sigma_{\tilde{c}_1,t} \quad (110)$$
$$=\gamma_1\sigma-\frac{\mu_1-\mu}{\sigma}+\gamma_1\tilde{c}_{2,t}\frac{\mathbf{R}_t}{\gamma_1\gamma_2}\left[(\sigma_{\xi,1}-\sigma_{\xi,2})+(\gamma_2-\gamma_1)\sigma\right]$$
$$=\mathbf{R}_t\sigma+\frac{\mu-\left(\frac{\tilde{c}_{1,t}}{\gamma_1}\mathbf{R}_t\mu_1+\frac{\tilde{c}_{2,t}}{\gamma_2}\mathbf{R}_t\mu_2\right)}{\sigma}$$
$$=\mathbf{R}_t\sigma+\frac{\mu-(w_{1,t}\mu_1+w_{2,t}\mu_2)}{\sigma}$$
$$=\mathbf{R}_t\sigma+\frac{\mu-\mathbf{u}}{\sigma}, \quad (111)$$

where in the fourth line we use the identity $w_{k,t}=\frac{\tilde{c}_{k,t}}{\gamma_k}\mathbf{R}_t$, and the fifth line we use the "aggregate belief" definition $\mathbf{u}$.

B. Drift Term: Interest rate

$$r_t + \frac{1}{2}\psi_t^2 = \left(\hat{r}_{1,t} + \frac{1}{2}\hat{\psi}_{1,t}^2\right) + \gamma_1\left(\mu_{\tilde{c}_1,t} - \frac{1}{2}\sigma_{\tilde{c}_1,t}^2\right) \tag{112}$$

$$r_t = \hat{r}_{1,t} + \gamma_1\mu_{\tilde{c}_1,t} - \gamma_1\hat{\psi}_{1,t}\sigma_{\tilde{c}_1,t} - \frac{\gamma_1(1+\gamma_1)}{2}\sigma_{\tilde{c}_1,t}^2 \tag{113}$$

Substituting every component:

$$r_t = \rho + \gamma_1\mu_1 - \lambda_x(\gamma_1 - 1)\omega_t - \frac{1}{2}\sigma^2\gamma_1(1+\gamma_1) \tag{114}$$

$$+ \frac{\tilde{c}_{2,t}}{\gamma_2}\mathbf{R}_t\Big\{-(\gamma_1-\gamma_2)\lambda_x(\bar{\omega}-\omega_t) \tag{115}$$

$$+ \frac{(\mu_2-\mu_1)}{\sigma^2}\left(\frac{\mu_1+\mu_2}{2} - \mu\right)$$

$$+ \frac{1}{2}\frac{\gamma_2 w_{2,t}^2 - \gamma_1 w_{1,t}^2}{\gamma_1\gamma_2}\left[\left(\frac{\mu_2-\mu_1}{\sigma}\right)^2 + 2(\mu_2-\mu_1)(\gamma_1-\gamma_2) + \sigma^2(\gamma_1-\gamma_2)^2\right]\Big\} \tag{116}$$

$$- \gamma_1(\gamma_1\sigma - \sigma_{\xi,1})\left(\frac{\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t(\sigma(\gamma_2-\gamma_1) + \sigma_{\xi,1} - \sigma_{\xi,2})\right) \tag{117}$$

$$- \frac{\gamma_1(1+\gamma_1)}{2}\left(\frac{\tilde{c}_{2,t}}{\gamma_1\gamma_2}\mathbf{R}_t(\sigma(\gamma_2-\gamma_1) + \sigma_{\xi,1} - \sigma_{\xi,2})\right)^2 \tag{118}$$

We consider the definition of $w_{2,t} = \tilde{c}_{2,t}\mathbf{R}_t/\gamma_2$.

Notice:

$$\gamma_2 w_{2,t}^2 - \gamma_1 w_{1,t}^2 \equiv \gamma_2 w_{2,t}\frac{\tilde{c}_{2,t}\mathbf{R}_t}{\gamma_2} - \gamma_1 w_{1,t}\frac{\tilde{c}_{1,t}\mathbf{R}_t}{\gamma_1} \equiv \mathbf{R}_t(\tilde{c}_{2,t} - w_{1,t}) \tag{119}$$

$$\mathbf{R}_t\mu_1 + w_{2,t}\gamma_1\sigma\sigma_\xi - w_{2,t}^2\sigma\sigma_\xi(\gamma_1-\gamma_2) \equiv \mathbf{R}_t(w_{1,t}\mu_1 + w_{2,t}\mu_2) \equiv \mathbf{R}_t\mathbf{u}_t$$

Then,

$$r_t = \rho - (\mathbf{R}_t - 1)\lambda_x\omega_t + \mathbf{R}_t\mathbf{u}_t - w_{2,t}w_{1,t}\sigma_\xi\sigma\mathbf{R}_t\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) + \frac{\sigma_\xi^2}{2}w_{1,t}w_{2,t}\left(1 - \frac{\mathbf{R}_t}{\gamma_1\gamma_2}\right)$$
$$\frac{\sigma^2}{2}(-\gamma_1^2 - \mathbf{R}_t) + w_{2,t}\frac{\sigma^2}{2}\frac{\mathbf{R}_t(\tilde{c}_{2,t} - w_{1,t})}{\gamma_1\gamma_2}(\gamma_1-\gamma_2)^2 - w_{2,t}\gamma_1\sigma^2(\gamma_2-\gamma_1) - \frac{(1+\gamma_1)}{2\gamma_1}w_{2,t}^2\sigma^2(\gamma_1-\gamma_2)^2$$

Notice that the following holds:

$$w_{2,t}(\gamma_1-\gamma_2) \equiv \gamma_1 - \mathbf{R}_t$$

Factorize $\mathbf{R}_t$:

$$r_t = \rho - (\mathbf{R}_t - 1)\lambda_x\omega_t + \mathbf{R}_t\mathbf{u}_t - w_{2,t}w_{1,t}\sigma_\xi\sigma\mathbf{R}_t\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) + \frac{\sigma_\xi^2}{2}w_{1,t}w_{2,t}\left(1 - \frac{\mathbf{R}_t}{\gamma_1\gamma_2}\right)$$
$$+ \frac{\sigma^2}{2}\mathbf{R}_t\left\{-\frac{\mathbf{R}_t}{\gamma_2} + \frac{\tilde{c}_{1,t}\mathbf{R}_t^2}{\gamma_1\gamma_2} - \frac{\tilde{c}_{1,t}\mathbf{R}_t^2}{\gamma_1^2} - \mathbf{R}_t\right\} \tag{120}$$

working on the second line (inside $\{\cdot\}$):

$$-\frac{\mathbf{R}_t}{\gamma_2} + \frac{\tilde{c}_{1,t}\mathbf{R}_t^2}{\gamma_1\gamma_2} - \frac{\tilde{c}_{1,t}\mathbf{R}_t^2}{\gamma_1^2} - \mathbf{R}_t \quad \equiv \quad -\mathbf{P}_t \tag{121}$$

Introducing expression (121) into (120)

$$r_t = \rho - (\mathbf{R}_t - 1)\lambda_x\omega_t + \mathbf{R}_t\mathbf{u}_t - w_{2,t}w_{1,t}\sigma_\xi\sigma\mathbf{R}_t\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) + \frac{\sigma_\xi^2}{2}w_{1,t}w_{2,t}\left(1 - \frac{\mathbf{R}_t}{\gamma_1\gamma_2}\right) + \frac{\sigma^2}{2}\mathbf{R}_t\left\{-\mathbf{P}_t\right\} \tag{122}$$

A.2 Proof of Lemma 2.

Use the definition of SDF

$$h_t^{(\tau)} = e^{-\rho\tau}\frac{e^{\gamma_1\omega_t + x_t}}{\xi_{1,t}\tilde{c}_{1,t}^{-\gamma_1}}E_t\underbrace{\left[e^{-(\gamma_1-1)\omega_{t+\tau}}\xi_{1,t+\tau}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}_{f(\omega_{t+\tau},\xi_{t+\tau},\xi_{1,t+\tau})} \tag{123}$$

where

$$\xi_{1,t+\tau} = \xi_{1,t}e^{-\frac{1}{2}\sigma_{\xi,1}^2\tau + \sigma_{\xi,1}\int_t^{t+\tau}dZ_s}, \tag{124}$$

$$\omega_{t+\tau} = \bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x\tau} + \sigma\int_t^{t+\tau}e^{-\lambda_x(t+\tau-s)}dZ_s. \tag{125}$$

The first part depends on state variables at t, which is straightforward computed since we discretize ω_t, x_t, and ξ_t (and hence we can calculate $\tilde{c}_{1,t}$). In the second part, we need to find a way to calculate $\tilde{c}_{1,t+\tau}$, which is a function of $\omega_{t+\tau}$ and $\xi_{t+\tau}$.

To compute $\tilde{c}_{1,t+\tau}$ directly from the risk-sharing rule, we need to express $\xi_{t+\tau}$ and $\omega_{t+\tau}$ as functions of $\tilde{s}$.

$$\xi_{t+\tau} = \xi_t e^{a_\tau + \frac{\mu_2-\mu_1}{\sigma}\int_t^{t+\tau}dZ_s}, \quad \text{with} \quad a_\tau = -\frac{1}{2}(\sigma_{\xi,2}^2 - \sigma_{\xi,1}^2)\tau \tag{126}$$

$$\omega_{t+\tau} = \bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x\tau} + \sigma\int_t^{t+\tau}e^{-\lambda_x(t+\tau-s)}dZ_s. \tag{127}$$

Therefore,

$$\xi_{t+\tau} = \xi_t e^{a_\tau + \frac{\mu_2-\mu_1}{\sigma}\sqrt{\tau}\times\tilde{s}}, \quad \text{with} \quad a_\tau = -\frac{1}{2}(\sigma_{\xi,2}^2 - \sigma_{\xi,1}^2)\tau \tag{128}$$

$$\omega_{t+\tau} = \bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x\tau} + \sigma e^{-\lambda_x\tau}\sqrt{\frac{e^{2\lambda_x\tau}-1}{2\lambda_x}} \times \tilde{s}. \tag{129}$$

Likewise,

$$\xi_{1,t+\tau} = \xi_{1,t}e^{-\frac{1}{2}\sigma_{\xi,1}^2\tau + \sigma_{\xi,1}\int_t^{t+\tau}dZ_s} \tag{130}$$

$$\xi_{1,t+\tau} = \xi_{1,t}e^{-\frac{1}{2}\sigma_{\xi,1}^2\tau + \sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}} \tag{131}$$

Thus, working on $h_t^{(\tau)}$,

$$
\begin{aligned}
h_t^{(\tau)} &= e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t}}{\xi_{1,t}\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}}\xi_{1,t+\tau}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right] \\
&= e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t}}{\xi_{1,t}\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}\xi_{1,t}e^{-\frac{1}{2}\sigma_{\xi,1}^2\tau+\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right] \qquad (132)\\
&= e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t}}{\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{-\frac{1}{2}\sigma_{\xi,1}^2\tau+\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right] \qquad (133)\\
&= e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t-\frac{1}{2}\sigma_{\xi,1}^2\tau}}{\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right] \qquad (134)\\
&= e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t-\frac{1}{2}\sigma_{\xi,1}^2\tau}}{\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right] \qquad (135)
\end{aligned}
$$

A.3 Proof of Lemma 3.

$$
\begin{aligned}
r_t^\tau &= \frac{1}{\tau}\log\left[\frac{e^{\omega_t+x_t}}{e^{-\rho\tau}\frac{e^{\gamma_1\omega_t+x_t-\frac{1}{2}\sigma_{\xi,1}^2\tau}}{\tilde{c}_{1,t}^{-\gamma_1}}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right]}\right] \qquad (136)\\
&= \frac{1}{\tau}\log\left[\frac{e^{\rho\tau-(\gamma_1-1)\omega_t+\frac{1}{2}\sigma_{\xi,1}^2\tau}\tilde{c}_{1t}^{-\gamma_1}}{E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right]}\right]\\
&= \frac{1}{\tau}\log\left[\frac{g(\omega_t,\tilde{c}_{1,t})}{E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}(\tilde{s})}e^{\sigma_{\xi,1}\sqrt{\tau}\times\tilde{s}}\left[\tilde{c}_{1,t+\tau}(\tilde{s})\right]^{-\gamma_1}\right]}\right] \qquad (137)
\end{aligned}
$$

A.4 Proof of Lemma 4.

1. From risk-sharing rule (??) and $cc_{k,t+\tau}\equiv\log(\tilde{c}_{,t+\tau})$ for $k\in\{1,2\}$, we have

$$e^{cc_{1,t+\tau}} = a^{-1/\gamma_1}\left(e^{cc_{2,t+\tau}}\right)^{\gamma_2/\gamma_1}e^{\omega_t(\gamma_2-\gamma_1)/\gamma_1}. \qquad (138)$$

The equilibrium condition in goods market becomes

$$e^{cc_{1,t+\tau}}+e^{cc_{2,t+\tau}}=1$$

We then use the following approximation in both equations:

$$e^{cc_{k,t+\tau}}\approx 1+cc_{k,t+\tau},$$

and after some algebra, we obtain the following approximate linear relationship

$$cc_{1,t+\tau}\approx -\frac{\log a+\gamma_2}{\gamma_1+\gamma_2}-\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\omega_{t+\tau}, \quad a=(1-\lambda)/\lambda, \quad \forall\tau\geq 0 \qquad (139)$$

2. $g(\omega_{t+\tau})$ is defined as

$$g(\omega_{t+\tau})) = e^{-(\gamma_1 - 1)\omega_{t+\tau} - \gamma_1 cc_{1,t+\tau}}$$

We then introduce equation (139) into $g(\omega_{t+\tau})$ and apply conditional expectation in t, we have

$$E_t\left[g(\omega_{t+\tau})\right] \approx e^{\gamma_1 \frac{\log a + \gamma_2}{\gamma_1 + \gamma_2} + b\mu_\tau + \frac{b^2}{2}\sigma_\tau^2} \tag{140}$$

$$\log E_t\left[g(\omega_{t+\tau})\right] \approx \gamma_1 \frac{\log a + \gamma_2}{\gamma_1 + \gamma_2} + b\mu_\tau + \frac{b^2}{2}\sigma_\tau^2, \tag{141}$$

where $\omega_{t+\tau} \sim N(\mu_\tau, \sigma_\tau^2)$ with μ_τ and σ_τ^2 defined by (**??**) and (**??**). The coefficient b is expressed as $b = (\gamma_1 + \gamma_2 - 2\gamma_1\gamma_2)/(\gamma_1 + \gamma_2) < 0$. We also use the property of normal distribution function: if $x \sim N(\mu, \sigma^2)$, $E[e^x] = e^{\mu + \frac{1}{2}\sigma^2}$.

3. Using expressions (39) and (**??**), the equity yields (Eq. **??**) can be expressed as follows:

$$\begin{aligned} r_t^\tau &= \rho - \frac{1}{\tau} z(\omega_t) - \frac{1}{\tau}\log E_t\left[g(\omega_{t+\tau})\right] \\ &= \rho - \frac{1}{\tau}\left(-\gamma_1 \frac{\log a + \gamma_2}{\gamma_1 + \gamma_2} - b\omega_t\right) - \frac{1}{\tau}\left(\gamma_1 \frac{\log a + \gamma_2}{\gamma_1 + \gamma_2} + b\mu_\tau + \frac{b^2}{2}\sigma_\tau^2\right) \\ &= \rho - \frac{1}{\tau}\left(-b\omega_t + b\mu_\tau + \frac{b^2}{2}\sigma_\tau^2\right) \end{aligned} \tag{142}$$

We then consider the expressions for μ_τ (Eq. **??**) and σ_τ (Eq. **??**) in Eq. (142):

$$\begin{aligned} r_t^\tau &= \rho + \frac{1}{\tau}\left(b(\omega_t - \mu_\tau) - \frac{b^2}{2}\sigma_\tau^2\right) \\ &= \rho + \frac{1}{\tau}\left[b(\omega_t - \bar{\omega})\left(1 - e^{-\lambda_x \tau}\right) - \frac{(b\sigma)^2}{4\lambda_x}\left(1 - e^{-2\lambda_x \tau}\right)\right] \end{aligned} \tag{143}$$

A.5 Proof of Lemma 5.

Substituting Eq. (50) into Eq, (51):

$$
\begin{aligned}
r_t^\tau &= \frac{1}{\tau}\log\left[\frac{e^{\omega_t+x_t}}{e^{-\rho\tau+x_t+\gamma_1\omega_t-\log\tilde{\xi}_{1,t}}\tilde{c}_{1t}^{\gamma_1}E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}+\log\tilde{\xi}_{1,t+\tau}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}\right], \\
&= \frac{1}{\tau}\log\left[\frac{e^{\rho\tau-(\gamma_1-1)\omega_t+\log\tilde{\xi}_{1,t}}\tilde{c}_{1t}^{-\gamma_1}}{E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}+\log\tilde{\xi}_{1,t+\tau}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}\right], \\
&= \rho+\frac{1}{\tau}\log\left[\frac{e^{-(\gamma_1-1)\omega_t+\log\tilde{\xi}_{1,t}}\tilde{c}_{1t}^{-\gamma_1}}{E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}+\log\tilde{\xi}_{1,t+\tau}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}\right] \\
&= \rho-\frac{1}{\tau}\log\left[\frac{E_t\left[e^{-(\gamma_1-1)\omega_{t+\tau}+\log\tilde{\xi}_{1,t+\tau}}\tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}{e^{-(\gamma_1-1)\omega_t+\log\tilde{\xi}_{1,t}}\tilde{c}_{1t}^{-\gamma_1}}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{-(\gamma_1-1)(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})-\gamma_1(\log\tilde{c}_{1,t+\tau}-\log\tilde{c}_{1,t})}\right]. \qquad (144)
\end{aligned}
$$

Next, $cc_{1,t+\tau} \equiv \log(\tilde{c}_{1,t+\tau})$ can be approximated in the same manner as shown in Eq. (36), with the key difference that the Radon-Nikodym derivative is now given by $\tilde{\xi}_t$ under diagnostic beliefs. Accordingly, $cc_{1,t+\tau}$ can be expressed as:

$$
cc_{1,t+\tau} \approx -A - B\omega_{t+\tau} - C\log\tilde{\xi}_{t+\tau}, \qquad (145)
$$

where the coefficients A, B, and C are defined as in Eq. Therefore,

$$
\begin{aligned}
\log\tilde{c}_{1,t+\tau}-\log\tilde{c}_{1,t} &\approx -A-B\omega_{t+\tau}-C\log\tilde{\xi}_{t+\tau}-(-A-B\omega_t-C\log\tilde{\xi}_t), \\
&\approx -B(\omega_{t+\tau}-\omega_t)-C(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t)
\end{aligned}
$$

Then, Eq. (144) becomes

$$
\begin{aligned}
r_t^\tau &= \rho-\frac{1}{\tau}\log E_t\left[e^{-(\gamma_1-1)(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})-\gamma_1(\log\tilde{c}_{1,t+\tau}-\log\tilde{c}_{1,t})}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{-(\gamma_1-1)(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})-\gamma_1(-B(\omega_{t+\tau}-\omega_t)-C(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t))}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{-(\gamma_1-1)(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})+\gamma_1(B(\omega_{t+\tau}-\omega_t)+C(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t))}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{[\gamma_1 B-(\gamma_1-1)](\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})+\gamma_1 C(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t)}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{b(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})+a(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t)}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{a(\log\tilde{\xi}_{t+\tau}-\log\tilde{\xi}_t)+b(\omega_{t+\tau}-\omega_t)+(\log\tilde{\xi}_{1,t+\tau}-\log\tilde{\xi}_{1,t})}\right] \\
&= \rho-\frac{1}{\tau}\log E_t\left[e^{\alpha}\right]. \qquad (146)
\end{aligned}
$$

where $a = \gamma_1/(\gamma_1+\gamma_2)$ and $b = (\gamma_1+\gamma_2-2\gamma_1\gamma_2)/(\gamma_1+\gamma_2) < 0$.

A.6 Proof of Lemma 6.

For simplicity, we bring the definition of the following variables from Eqs. (32), (45), and (47):

$$
\begin{aligned}
\tilde{\xi}_t &= \exp\left(-\frac{1}{2}\int_0^t \left[\sigma^2_{\tilde{\xi},2}(\omega_s) - \sigma^2_{\tilde{\xi},1}(\omega_s)\right] ds + \int_0^t \sigma_{\tilde{\xi}}(\omega_s)\, dZ_s\right), \\
\tilde{\xi}_{k,t} &= \exp\left(-\frac{1}{2}\int_0^t \sigma^2_{\tilde{\xi},k}(\omega_s)\, ds + \int_0^t \sigma_{\tilde{\xi},k}(\omega_s)\, dZ_s\right), \\
\omega_{t+\tau} &= \bar{\omega} + (\omega_t - \bar{\omega})e^{-\lambda_x \tau} + \sigma e^{-\lambda_x \tau}\sqrt{\frac{e^{2\lambda_x \tau} - 1}{2\lambda_x}} \times \tilde{s},
\end{aligned}
$$

where:

$$
\begin{aligned}
\sigma_{\tilde{\xi},k} &= \frac{\theta_k}{\sigma}(\omega_t - \bar{\omega}), \\
\sigma_{\tilde{\xi}} &= \sigma_{\tilde{\xi},2} - \sigma_{\tilde{\xi},1} = \frac{\theta_2 - \theta_1}{\sigma}(\omega_t - \bar{\omega}).
\end{aligned}
$$

We begin with $\log\tilde{\xi}_t$:

$$
\begin{aligned}
\log\tilde{\xi}_t &= -\frac{1}{2}\int_0^t \left[\sigma^2_{\tilde{\xi},2} - \sigma^2_{\tilde{\xi},1}\right] ds + \int_0^t \sigma_{\tilde{\xi}}\, dZ_s, \\
\log\tilde{\xi}_{t+\tau} &= -\frac{1}{2}\int_0^{t+\tau} \left[\sigma^2_{\tilde{\xi},2} - \sigma^2_{\tilde{\xi},1}\right] ds + \int_0^{t+\tau} \sigma_{\tilde{\xi}}\, dZ_s.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\log\tilde{\xi}_{t+\tau} - \log\tilde{\xi}_t &= -\frac{1}{2}\int_t^{t+\tau} \left[\sigma^2_{\tilde{\xi},2} - \sigma^2_{\tilde{\xi},1}\right] ds + \int_t^{t+\tau} \sigma_{\tilde{\xi}}\, dZ_s, \\
&= -\frac{1}{2}\int_t^{t+\tau} \left[\frac{\theta_2^2}{\sigma^2}(\omega_s - \bar{\omega})^2 - \frac{\theta_1^2}{\sigma^2}(\omega_s - \bar{\omega})^2\right] ds + \int_t^{t+\tau} \frac{\theta_2 - \theta_1}{\sigma}(\omega_s - \bar{\omega})\, dZ_s, \\
&= -\frac{1}{2}\left[\frac{\theta_2^2}{\sigma^2} - \frac{\theta_1^2}{\sigma^2}\right]\int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds + \frac{\theta_2 - \theta_1}{\sigma}\int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s, \\
a(\log\tilde{\xi}_{t+\tau} - \log\tilde{\xi}_t) &= -\frac{a}{2}\left[\frac{\theta_2^2 - \theta_1^2}{\sigma^2}\right]\int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds + a\left[\frac{\theta_2 - \theta_1}{\sigma}\right]\int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s.
\end{aligned}
$$

Similarly, for $\tilde{\xi}_{k,t}$ we have:

$$
\begin{aligned}
\log\tilde{\xi}_{1,t+\tau} - \log\tilde{\xi}_{1,t} &= -\frac{1}{2}\int_t^{t+\tau} \sigma^2_{\tilde{\xi},1}\, ds + \int_t^{t+\tau} \sigma_{\tilde{\xi},1}\, dZ_s, \\
&= -\frac{1}{2}\int_t^{t+\tau} \frac{\theta_1^2}{\sigma^2}(\omega_s - \bar{\omega})^2\, ds + \int_t^{t+\tau} \frac{\theta_1}{\sigma}(\omega_s - \bar{\omega})\, dZ_s, \\
&= -\frac{1}{2}\frac{\theta_1^2}{\sigma^2}\int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds + \frac{\theta_1}{\sigma}\int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s,
\end{aligned}
\tag{147}
$$

Then,

$$
\begin{aligned}
a(\log \tilde{\xi}_{t+\tau} - \log \tilde{\xi}_t) + \log \tilde{\xi}_{1,t+\tau} - \log \tilde{\xi}_{1,t} &= \left\{ -\frac{1}{2}\frac{\theta_1^2}{\sigma^2} - \frac{a}{2}\left[\frac{\theta_2^2 - \theta_1^2}{\sigma^2}\right]\right\} \int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds \\
&\quad + \left\{ \frac{\theta_1}{\sigma} + a\left[\frac{\theta_2 - \theta_1}{\sigma}\right]\right\} \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s, \\
&= -\frac{1}{2\sigma^2}\left\{\theta_1^2 + a\left[\theta_2^2 - \theta_1^2\right]\right\} \int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds \\
&\quad + \frac{1}{\sigma}\left\{\theta_1 + a\left[\theta_2 - \theta_1\right]\right\} \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s, \\
&= -\frac{(1-a)\theta_1^2 + a\theta_2^2}{2\sigma^2} \int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds \\
&\quad + \frac{(1-a)\theta_1 + a\theta_2}{\sigma} \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s, \\
&= \alpha_0 \int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds + \alpha_1 \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s,
\end{aligned}
$$

where:

$$
\alpha_0 = -\frac{(1-a)\theta_1^2 + a\theta_2^2}{2\sigma^2}, \qquad \alpha_1 = \frac{(1-a)\theta_1 + a\theta_2}{\sigma}.
$$

Next, for $s \in [0, t+\tau]$:

$$
\begin{aligned}
d\omega_s &= \lambda_x(\bar{\omega} - \omega_s)\, ds + \sigma\, dZ_s, \\
\int_t^{t+\tau} d\omega_s &= \int_t^{t+\tau} \lambda_x(\bar{\omega} - \omega_s)\, ds + \int_t^{t+\tau} \sigma\, dZ_s, \\
\omega_{t+\tau} - \omega_t &= \int_t^{t+\tau} \lambda_x(\bar{\omega} - \omega_s)\, ds + \int_t^{t+\tau} \sigma\, dZ_s, \\
b(\omega_{t+\tau} - \omega_t) &= -b\lambda_x \int_t^{t+\tau} (\omega_s - \bar{\omega})\, ds + b\sigma \int_t^{t+\tau} dZ_s.
\end{aligned}
$$

Then, α is:

$$
\alpha = \alpha_0 \int_t^{t+\tau} (\omega_s - \bar{\omega})^2\, ds + \alpha_1 \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s - b\lambda_x \int_t^{t+\tau} (\omega_s - \bar{\omega})\, ds + b\sigma \int_t^{t+\tau} dZ_s
$$

B Numerical Integration Method

In this section, we describe the implementation of Gaussian quadrature procedure.

Step 1. We define $\omega_{t+\tau} \sim N(\mu_\tau, \sigma_\tau^2)$

Step 2. We calculate

$$[x, \text{weights}] = \text{qnwnorm}(n, \mu_\tau, \sigma_\tau^2),$$

where x is a vector of $\omega_{t+\tau}$ for each point of the grid (n points):

$$x = \begin{bmatrix} \omega_{t+\tau,1} \\ \omega_{t+\tau,2} \\ \vdots \\ \omega_{t+\tau,n} \end{bmatrix} \tag{148}$$

Step 3. We calculate $\tilde{c}_{1,t+\tau}$ for each value of x ($x_i = \omega_{t+\tau,i}$) using the risk-sharing rule:

$$e^{-\gamma_1 x_i} \left(\tilde{c}_{1,t+\tau,i}\right)^{-\gamma_1} = a \left(1 - \tilde{c}_{1,t+\tau,i}\right)^{-\gamma_2} e^{-\gamma_2 x_i}$$

Step 4. We define the function inside of the expectation operator: $f(x_i)$

$$f(x_i) = e^{-(\gamma_1 - 1)x_i} \left(\tilde{c}_{1,t+\tau,i}\right)^{-\gamma_1}$$

Then $f(x)$ is a column vector with element equals to $f(x_i)$.

Step 5. We calculate the expectation:

$$E\left[f(x)\right] = \text{weights}' \times f(x)$$

C Monte Carlo Simulation

In this section, we describe the implementation of Monte Carlo procedure for evaluating $\mathbb{E}_t\left[e^{\alpha}\right]$, where process α is defined as:

$$\alpha = \alpha_0 \int_t^{t+\tau} (\omega_s - \bar{\omega})^2 ds + \alpha_1 \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s - b\lambda_x \int_t^{t+\tau} (\omega_s - \bar{\omega}) ds + b\sigma \int_t^{t+\tau} dZ_s.$$

We proceed in four steps:

- **Step 1: Discretize the time interval $[t, t+\tau]$.** Let N be the number of time steps, and define the time increment as:

$$\Delta t = \frac{\tau}{N}.$$

- **Step 2: Simulate sample paths of ω_s.** We simulate M independent paths of the process ω_s over $[t, t+\tau]$, which follows a OU process, using the Euler–Maruyama discretization:

$$\omega_{i+1} = \omega_i + \lambda_x(\bar{\omega} - \omega_i)\Delta t + \sigma\sqrt{\Delta t} \cdot \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0,1), \quad \text{for } i = 0,$$

with initial condition $\omega_0 = \omega_t$. The path of ω_s is given by:

$$[\omega_0, \omega_1, \omega_2, \ldots, \omega_N] = [\omega_t, \omega_{t+\Delta t}, \omega_{t+2\Delta t}, \ldots, \omega_{t+N\Delta t}].$$

- **Step 3: Approximate the integrals along each path.** For each simulated path, approximate the components of α as:

$$A = \int_t^{t+\tau} (\omega_s - \bar{\omega})^2 ds \approx \sum_{i=0}^{N-1} (\omega_i - \bar{\omega})^2 \Delta t,$$
$$B = \int_t^{t+\tau} (\omega_s - \bar{\omega})\, dZ_s \approx \sum_{i=0}^{N-1} (\omega_i - \bar{\omega}) \cdot \sqrt{\Delta t} \cdot \varepsilon_i,$$
$$C = \int_t^{t+\tau} (\omega_s - \bar{\omega}) ds \approx \sum_{i=0}^{N-1} (\omega_i - \bar{\omega}) \Delta t,$$
$$D = \int_t^{t+\tau} dZ_s \approx \sum_{i=0}^{N-1} \sqrt{\Delta t} \cdot \varepsilon_i.$$

Then, for simulation m, we compute:

$$\alpha^{(m)} = \alpha_0 A + \alpha_1 B - b\lambda_x C + b\sigma D,$$

- **Step 4: Compute the Monte Carlo estimate.** The Monte Carlo estimator of $\mathbb{E}_t[e^{\alpha}]$ is given by:

$$\mathbb{E}_t\left[e^{\alpha}\right] \approx \frac{1}{M} \sum_{m=1}^{M} \exp\left(\alpha^{(m)}\right).$$

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